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A stochastic schedule-following simulation model of bus routes

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ABSTRACT

Microsimulation models of bus routes allow transit operators to both better understand the dynamics of bus routes and facilitate better policy making. Several simulation models of bus routes have been proposed in the literature, including cellular-automata, bus-following and traffic-following models. The majority of these approaches aim to simulate the interactions of a bus with other buses (the bus-following model), with passengers or the surrounding traffic (the traffic-following model), but they all fail to consider the important interactions between buses and their schedules. In a conventional schedule-based public transport system, bus drivers aim to arrive at each stop on time. This means that they will either speed up or slow down if their vehicles are not meeting the schedule. The research within this paper is a novel contribution to the literature of bus route simulation. We introduce the first schedule-following model where buses try to adhere to their schedule in a conventional schedule-based public transport system. A simulated numerical analysis shows the characteristics of the proposed schedule-following model and compares it to existing models. Finally, the model is calibrated using Automatic Vehicle Location and Smart Card data from Brisbane, Australia. The results show good model performance against the observed data. The model is relatively simple, yet the fundamental mechanisms that drive the model are novel and it has the potential to be applied in any city with well-defined bus schedules.

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1. Introduction

Microsimulation of transport systems is a topic of considerable interest from researchers and practitioners. Traffic microsimulation in particular has evolved from simple car-following models (Tang et al. 2012b) to sophisticated commercial software packages such as Aimsun (Aimsun 2018). Public transport simulation is often reduced to a simple component in these packages, with the main focus being to investigate the impact of buses on traffic.

Nevertheless, the simulation of buses has attracted several methodological approaches over the years with Cellular Automata (CA) modelling being one of the most successful approaches (Luo et al. 2012; O'loan, Evans, and Cates 1998; Chowdhury and Desai 2000; Jiang, Bin Jia, and Wu 2003). Whilst the dynamical foundations of CA models are well understood, they are regularly outperformed by more sophisticated models such as bus-following models (Nagatani 2000; Huijberts 2002; Tang et al. 2012a; Nagatani 2001; Hill 2003) and traffic-following models (Cats et al. 2010; Toledo et al. 2010; Hans et al. 2015). Bus-following models account for the fundamental dynamics of a bus route as individual buses follow each other. They are similar to car-following models in that the speed of the buses is dynamically adjusted to maintain the schedule. Traffic-following models take a more holistic approach

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simulating buses as one component of the transport system (private and public vehicles are also accounted for), where their speeds are affected by the traffic flow, traffic signals (Hans et al. 2015) or the density of the surrounding traffic (Toledo et al. 2010).

In general, bus-following models focus on the interactions between buses, while traffic-following models focus on the interactions between buses and the broader traffic system. While the bus schedule exists in both bus-following and traffic-following models, they do not attempt to capture the *schedule-following behaviour* of buses. In a conventional bus schedule, drivers aim to visit each of their stops at a set time (Chen, Liu, and Xia 2005). If they are behind schedule, they adjust their speed accordingly. This paper presents the first bus simulation that both captures the schedule-following mechanism, while also being able to account for other important phenomena (as observed from the Automatic Vehicle Location (AVL) data). The theoretical properties of the model include: (1) individual (bus) drivers whose aim is to meet their own schedules; (2) *bus bunching* that occurs when the passenger demand is large and buses are delayed while passengers embark and (3) vehicle overtaking and *leapfrogging* that occurs alongside bus bunching. We define bus bunching as the situation when two or more buses arrive at the same bus stop at the same time. Leapfrogging is a special variant of bus bunching that occurs when two vehicles cannot separate from each other over multiple stops, e.g. one bus overtakes another and is later overtaken again by the same bus.

The main contribution of this paper is the methodological development of the schedule-following bus simulation model. This innovative approach is one of the first successful models that is capable of accounting for the combination of schedule following, bus bunching and leapfrogging. The paper also provides a numerical analysis using synthetic data and a calibration of the model using observed AVL and Smart Card data. Whilst there are clear limitations to the work as a transport planning/management tool, for example the absence of traffic or a realistic transport network, the fundamental mechanisms that drive the model are novel and can be incorporated with other mechanisms such as traffic-following behaviour to create a more realistic simulation of bus operation.

The paper is structured as follows. Section 2 reviews the current state of the art in the bus simulation literature. Section 3 details the motivation to replicate the actual dynamics of buses as observed from AVL data. Section 4 describes the model development process. Section 5 provides some numerical results of the proposed model in comparison with existing models in the literature. Section 6 illustrates the calibration of the proposed model. Finally, Section 7 concludes the study, summarises the contributions and suggests some future research directions.

2. Literature review

Models of bus operations are commonly used to predict future system states and to simulate control strategies (Eberlein et al. 1998; Hickman 2001; Sánchez-Martínez, Koutsopoulos, and Wilson 2016). The most popular model type is static, where travel time or travel speed on each link is assumed to be deterministic (Eberlein et al. 1998). It is also commonly assumed that bus travel time follows a probabilistic distribution (Daganzo 2009). A recent paper by Sánchez-Martínez, Koutsopoulos, and Wilson (2016) proposes a dynamic factor function to translate the static bus travel time and demand into time-dependent functions to be considered in bus holding control strategies. Although those models do not yet consider flow dynamics under different system states, some mathematical models have attempted to simulate interactions between other buses, with passengers and with the surrounding traffic. These latest advances can be classified into CA models (Luo et al. 2012; O'loan, Evans, and Cates 1998; Chowdhury and Desai 2000; Jiang, Bin Jia, and Wu 2003), bus-following models (Nagatani 2000; Huijberts 2002; Tang et al. 2012a; Nagatani 2001; Hill 2003) and traffic-following models (Toledo et al. 2010; Hans et al. 2015; Cats et al. 2010).

CA models use discrete variables to model the dynamical properties of the bus system (O'loan, Evans, and Cates 1998). The road network is generally divided into a regular grid of cells in a discrete one-dimension lattice, where each cell can take a binary state (1 or 0) representing the presence of a bus on the cell, while time is discretised into fixed time steps. CA models generally aim to describe the dynamics of bus operations from rule-based local behaviours of individual buses: acceleration, deceleration, stopping and running, without many interactions between multiple vehicles. CA models have been adapted for simulating bus route operations (O'loan, Evans, and Cates 1998; Chowdhury and Desai 2000) and have recently been enhanced by incorporating vehicle capacity (Jiang, Bin Jia, and Wu 2003) and open boundary conditions (Luo et al. 2012). While CA models are simple to implement and efficient in performance, they rely on a discretisation of the continuous spatial and temporal spaces. They also aim to model the bus operation from the dynamics of individual vehicles rather than emphasising the flow dynamics of the system.

Taking inspiration from car-following models, e.g. Tang et al. (2012b), bus-following models (Nagatani 2000; Huijberts 2002; Tang et al. 2012a; Nagatani 2001; Hill 2003) are built on the interactions between multiple buses, in particular on the logic that buses follow each other. The first bus-following models (Nagatani 2000; Huijberts 2002; Tang et al. 2012a) modified the car-following velocity function from Newell (1961) and Whitham (1990) to capture the behaviour of bus drivers, who usually speed up when the headway (or time gap) between buses is large and slow down otherwise. An additional term in the optimal velocity function was introduced to represent the passenger boarding time at a bus stop. The bus-following model was later modified to become a time-headway model (Nagatani 2001; Hill 2003), where a desired headway was introduced instead of the optimal velocity. These models are the best representation of a highly frequent system, where drivers try to maintain a regular headway with leading buses. However, the follow-the-leader logic means that the well-known bus bunching phenomenon (Sun and Schmöcker 2018) is treated similarly to a traffic crash and will never occur in the model.

Traffic-following models of bus routes often derive the dynamics of bus operations from the interaction with the surrounding traffic, under the rationale that buses are a part of the overall traffic flow. These models usually separate a link model, where buses follow the traffic, and a node model where buses dwell for passengers boarding and alighting. Cats et al. (2010) developed their model as a component of a traffic-following simulation model. The model of Toledo et al. (2010) estimated a bus average speed based on the current, maximum and minimum traffic density. At stops, passenger arrivals follow a Poisson distribution and passenger alighting follows a Binomial distribution. Hans et al. (2015) explicitly estimated bus travel time based on traffic signals and traffic flows. The dwell time and the number of passengers are stochastically generated from Exponential or Poisson distributions. While assuming that the travel time on links is deterministic, Fonzone, Schmöcker, and Liu (2015) introduced the non-uniformity dynamics of passenger arrival to bus stops and concluded that the arrival patterns can worsen or improve bus bunching. Various traffic microsimulation packages, such as Aimsun (Barcel and Casas 2005), Vissim (Fellendorf and Vortisch 2010) and SUMO (Behrisch et al. 2011), also aim to include buses in traffic simulation.

However, while interactions between multiple buses (e.g. bus following behaviour in bus-following models), between buses and surrounding traffic (e.g. traffic-following or traffic microsimulation models) and between buses and passengers (all models) are all well studied, this is not the case for the interactions between buses and their schedules. Chen, Liu, and Xia (2005) proposed a bus arrival prediction method based on the Kalman Filter algorithm where the driver 'schedule recovery' behaviour is considered. Schedule recovery was defined as the effort from bus drivers to adhere to the schedule, similar to the schedule-following behaviour described in this paper. Chen, Liu, and Xia 2005 found empirically that schedule recovery behaviour could be observed in half of the bus trips in North Eastern United States. Ji, He, and Zhang (2014) measured bus drivers' schedule-following behaviours and their impacts on bus reliability using automatic vehicle location data. Wu, Liu, and Jin (2018) developed a bus holding control at stops that incorporates drivers' schedule recovery behaviour.

The bus route model to be proposed in this paper departs from the existing literature by capturing the 'schedule-following' dynamics of the bus under a conventional scheduled-based system in a microsimulation model of a bus route. The proposed model aims to reproduce the actual dynamics of the bus route, as observed from bus trajectories in real AVL data.

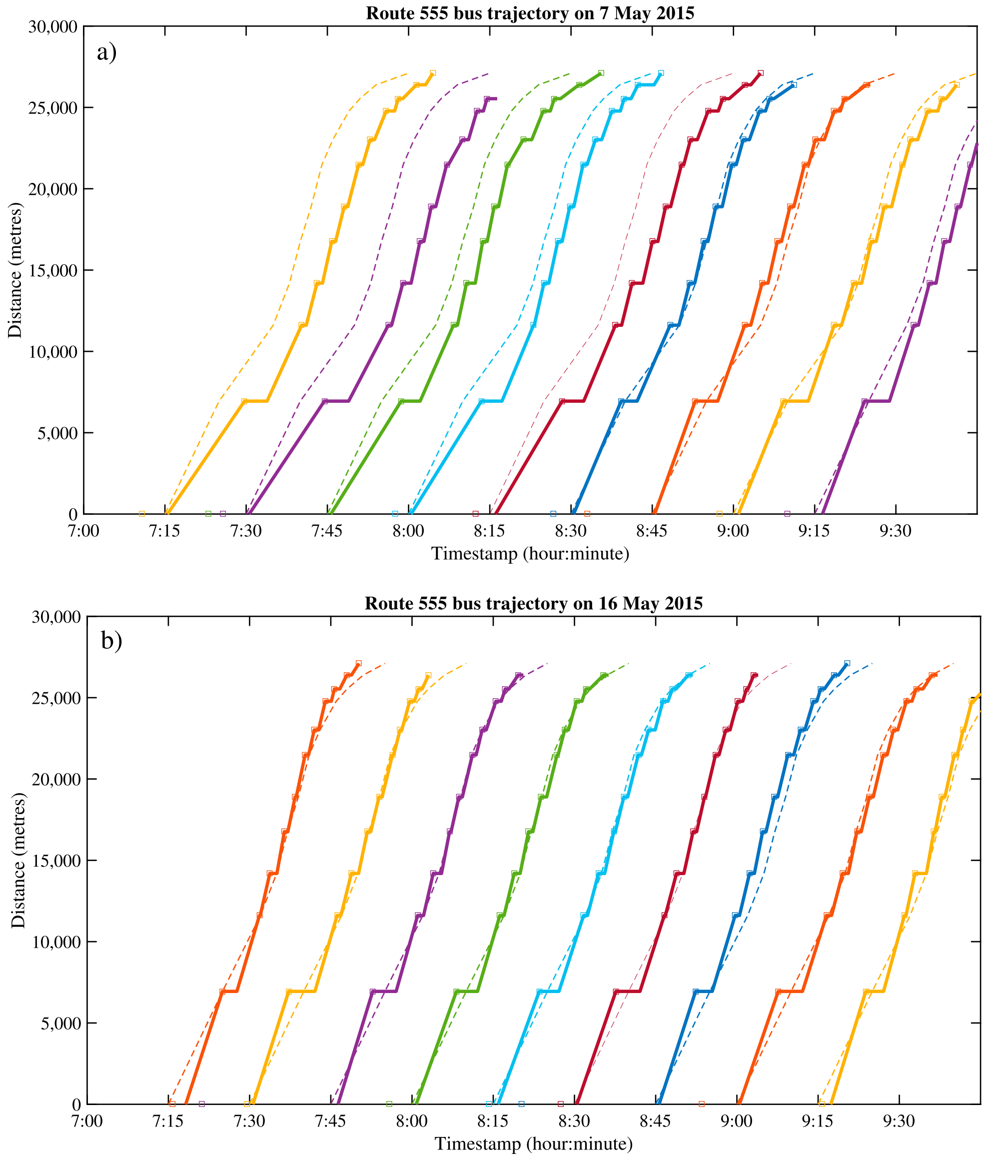

Figure 1. Observed bus trajectories (bold line) compared with the schedule (dashed line) from (a) weekday and (b) weekend.

3. Problem description

Figure 1 shows the observed trajectories of buses on Route 555 in Brisbane, Australia, on a typical weekday and a weekend. Route 555 is a busy 12-stop bus route connecting Loganholme to Brisbane CBD. Buses on Route 555 operate on segregated busways, so there is no impact of the surrounding traffic on the bus operation. The AVL data include the scheduled (dashed line) and actual (bold line) arrival and departure times of each bus.

Route 555 operates with the same scheduled travel time of 45 minutes and scheduled headway of 15 minutes during both weekdays and weekends. Figure 1(b) shows that during weekends, drivers are able to maintain good on-time performance at the majority of stops. It shows that drivers try to adhere to the schedule without any dynamic control. However, there is a heavy delay during weekdays (1a). Recall that Route 555 operates on a busway, this delay be the result of excessive passenger demand.

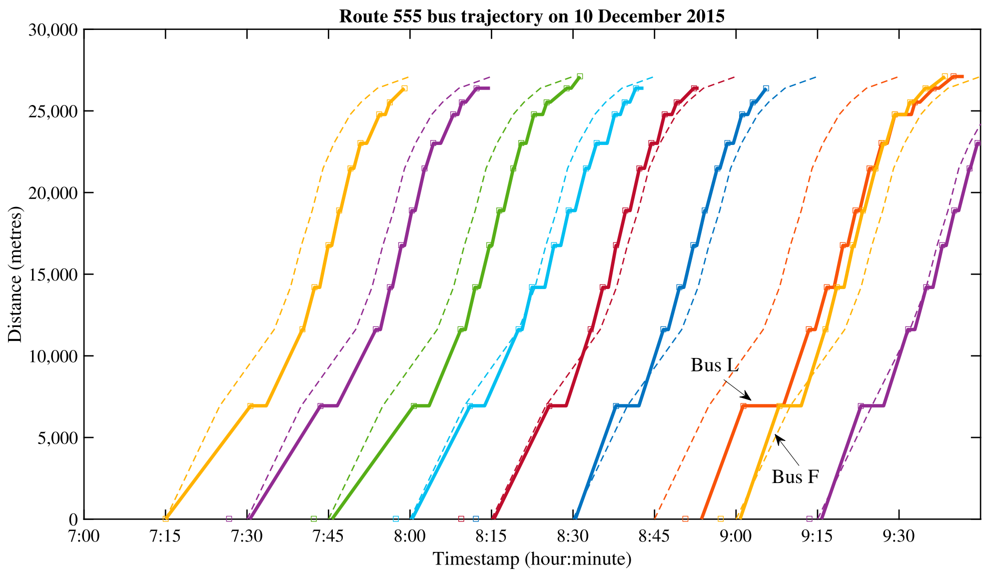

Figure 2. Observed bus bunching at Route 555.

Bus bunching is also a problem on high frequency bus routes such as Route 555. Figure 2 shows an example of the phenomenon. When bus L is late, the following bus (bus F) tends to travel faster due to having lower passenger demand. Both bus bunching and leapfrogging can be clearly observed in this situation. After the two buses are bunched, bus F overtakes bus L, resulting in the two buses being unable to separate from each other. As bus F overtakes bus L, it faces a heavy passenger demand awaiting for bus L. Bus F may now be the slow one due to this heavy demand and may be overtaken by bus L at downstream stops. This paper aims to capture these dynamics, which are classified as

- Follow-the-schedule: Buses try to adhere to schedules as much as they can
- Bus bunching: Bunching may occur when a bus is late and the following bus catches up with it
- Leap-frogging: As bus bunching occurs, the two bus cannot separate from each other.

While bus bunching has been discussed in the literature (Abkowitz and Tozzi 1987; Fonzone, Schmöcker, and Liu 2015), existing studies fail to model the follow-the-schedule dynamics of buses. A model that replicates the actual dynamics of the bus system will provide a better understanding of the bus operations and a better simulation of bus routes.

4. Schedule-following model formulation

This section aims to propose the schedule-following bus route model that captures the three aforementioned dynamics.

4.1. Notation

N:	Number of buses
j:	Index of vehicle ($j = 1 \ldots N$)
m:	Index of bus stop ($m = 1 \ldots M$)
M:	Number of bus stops
ω:	Slack time
H:	Scheduled headway

C:	Bus capacity
L_{m-1}:	The distance between stop $m-1$ and stop m
$v_{\max}$:	Maximum mean speed
$v_{\min}$:	Minimum mean speed
$t^a_{j,m}$:	Arrival time of bus j at stop m
$t^d_{j,m}$:	Departure time of bus j from stop m
$\delta t_{j,m}$:	Time headway or difference between the departure time of two adjacent buses from the same stop m
$D_{j,m}$:	Dwell time of bus j at stop m for passenger boarding and alighting
$\theta_1, \theta_2, \theta_3$:	Parameters set for estimating $D_{j,m}$
$\Delta_{j,m}$:	Time difference between actual and scheduled departure time of bus j at stop m
$\tau_{j,m}$:	Scheduled departure time of bus j from stop m
$B_{j,m}$:	Number of boarding passengers to bus j at stop m
$A_{j,m}$:	Number of alighting passengers from bus j at stop m
$Occ_{j,m}$:	Occupancy of bus j leaving stop m
$I_m(t)$:	Number of passengers waiting at stop m at time t

4.2. Assumptions

To develop the follow-the-schedule bus route model, the following assumptions are made:

- The bus fleet is homogeneous with similar capacity.
- Buses are allowed to freely overtake each other when possible: this assumption is to make sure that the leapfrogging phenomenon can be reproduced.
- Buses have two doors to allow simultaneous boarding and alighting.

4.3. Model formulation

We consider a general bus route on an one-dimensional lattice with open boundary conditions, as illustrated in Figure 3. Each bus travels from Stop 1 to M. The distance between stop $m-1$ and m is L_{m-1}. The arrival time $t^a_{j,m}$ of bus j to stop m is a function of the departure time $t^d_{j,m-1}$ from stop $m-1$ and the travel time between stop $m-1$ and stop m:

$$t^a_{j,m} = t^d_{j,m-1} + \frac{L_{m-1}}{v_{j,m-1}}, \tag{1}$$

where $v_{j,m-1}$ is the mean speed of bus j ($j \in [1 \ldots N]$) between stops $m-1$ and m. A bus driver operates the bus according to its lateness/earliness to the schedule at the previous stop, where the schedule adherence information is given to the driver each time the bus reaches a bus stop. If the bus is behind the departure schedule at the previous stop, the driver will operate with high mean speed, and otherwise with low mean speed when ahead of the schedule. We assume that this mean speed is dependent on the schedule adherence $\Delta_{j,m-1}$ at the last visited stop $m-1$:

$$v_{j,m-1} = V(\Delta_{j,m-1}). \tag{2}$$

We modify the formulation of the optimal speed function $V(\Delta_{j,m-1})$ from the bus-following model proposed in Nagatani (2001) and later discussed in Hill (2003). This optimal speed function is different to those in the bus-following models (Nagatani 2000, 2001; Huijberts 2002). These models use a modified optimal speed function that is similar to that used in a car-following model. Thus the bus dynamics are based on the distance between them (time or space headway) and a predefined critical headway. The proposed schedule-following model, instead, uses the ratio between *schedule adherence* $\Delta_{j,m}$ and scheduled headway H to define the speed of buses. In general, buses in the proposed

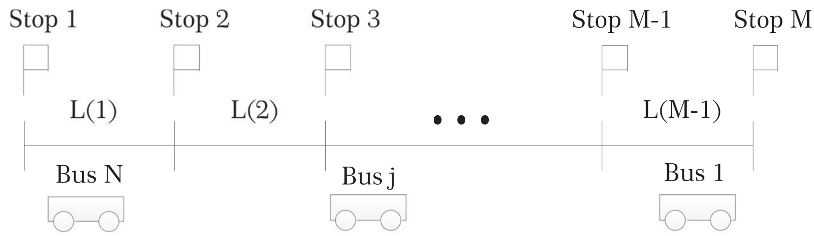

Figure 3. Schematic illustration of the bus route

schedule-following model do not 'see' other buses but only operate according to its own schedule. This formulation uses the same number of variables as in Nagatani (2001) and Hill (2003). $V(\Delta_{j,m-1})$ can be calculated in the proposed schedule-following model as follows:

$$V(\Delta_{j,m}) = v_{\min} + (v_{\max} - v_{\min})\frac{\tanh(\Delta_{j,m}/H) + 1}{2}, \tag{3}$$

where $v_{\min}$ and $v_{\max}$ are the minimum and maximum mean speeds, respectively. H is the scheduled headway between buses. Parameters $v_{\min}$ and $v_{\max}$ can be defined as deterministic or traffic-dependent variables, where the bus cannot travel slower or faster than the surrounding traffic. Although the data being used in this project are from a busway system with segregated right of way, these parameters are included to make the model more generalisable. Practitioners may incorporate up-to-date traffic data to dynamically calibrate these parameters using *data assimilation* methods, such as a Bayesian Filter (Kalnay 2003) or a Particle Filter (Kieu, Malleson, and Heppenstall 2019). In this paper, parameter calibration is performed offline to adjust the values of $v_{\min}$ and $v_{\max}$ to minimise the difference between estimated velocity $V(\Delta_{j,m-1})$ and the observed velocity. We will discuss the parameter calibration in Section 6.

The hyperbolic tangent factor is a smooth, spread out function that is used to vary the value of $V(\Delta_{j,m})$ between $v_{\min}$ (when the bus is ahead of the schedule) and $v_{\max}$ (when the bus is behind the schedule). The schedule adherence $\Delta_{j,m}$ is defined as the difference between the actual departure time $t^d_{j,m}$ and scheduled departure time $\tau_{j,m}$

$$\Delta_{j,m} = t^d_{j,m} - \tau_{j,m}. \tag{4}$$

In this paper, $\tau_{j,m}$ is calculated using the minimum time it takes to travel between stops $m-1$ and m, plus a certain amount of slack time ω to accommodate for dwell time and other uncertainties. Parameter ω is a part of the network settings, similar to parameters L, M and N.

$$\tau_{j,m} = \tau_{j,m-1} + \frac{L_{m-1}}{v_{\max}} + \omega. \tag{5}$$

The departure time of bus j from stop m is calculated from the arrival time $t^a_{j,m}$ plus the time spent at stops for passenger boarding and alighting, or in other words the dwell time D_m.

$$t^d_{j,m} = t^a_{j,m} + D_{j,m}. \tag{6}$$

$D_{j,m}$ is calculated as a function of the number of boarding and alighting passengers:

$$D_{j,m} = \theta_1 + \max\{\theta_2 \times B_{j,m}, \theta_3 \times A_{j,m}\}, \tag{7}$$

where $B_{j,m}$ and $A_{j,m}$ are the number of boarding and alighting passengers to bus j at stop m. The parameter set $[\theta_1, \theta_2, \theta_3]$ represents fixed values for vehicle stopping and starting (θ_1) and the time spent for passenger boarding (θ_2) and alighting (θ_3). For a single door bus system, where passengers alight first then board the bus, the following equation can be used:

$$D_{j,m} = \theta_1 + \theta_2 \times B_{j,m} + \theta_3 \times A_{j,m}. \tag{8}$$

These formulations of the dwell time are consistent with many studies in the literature, such as Bertini and El-Geneidy (2004) and the TCQSM (TRB 2013).

The number of alighting passenger can be estimated as

$$A_j(m)) = Occ_j(m-1) \times \rho_m, \tag{9}$$

where $Occ_j(m-1)$ is the occupancy or the number of passenger inside bus j prior to stop m. ρ_m is the probability of a random passenger to alight at stop m.

The number of boarding passengers $B_{j,m}$ is stochastic because passenger arrivals at bus stops are random. $B_{j,m}$ depends on the time gap between arrival time $t^a_{j,m}$ and departure time of the last vehicle $t^d_{j-1,m}$ visiting stop m. The passenger arrival process is modelled as a homogeneous Poisson process with an average arrival rate λ_m. The use of the Poisson process to model passenger arrivals is consistent with many previous studies (Toledo et al. 2010). The passenger arrival process is random and independent from the process of vehicle arrivals, which can be expressed in the following Algorithm 1.

Algorithm 1: Poisson arrival process

```
Set M                                                        ▷ Number of stops
Initialise Simulation_step=0.1                               ▷ Set the simulation step size
Initialise Events = Total_study_time / Simulation_step       ▷ Initialise the total possible events
for s=1 to M do
                                                             ▷ Loop from the first to the last stop
    arrival_probability = Random[0, 1]
                           ▷ A passenger will arrive if the arrival probability is large enough
    Set Events(s) = (arrival_probability > λ × Simulation_step)
end
return Events
```

Here, *Events* is a matrix of passengers with arrival times at each stop. We define $I_m(t)$ as the count of available passengers in *Events* who have arrived at stop m at time t and have not yet boarded any bus. The number of boarding passengers $B_{j,m}$ is the minimum between the number of available passengers, who arrived at stop m before time $t^a_{j,m}$, and the residual capacity of bus j after alighting $C + A_{j,m} - Occ_{j,m}$, where C is the capacity

$$B_{j,m} = \min\{I_m(t^a_{j,m}), C + A_{j,m} - Occ_{j,m}\}. \tag{10}$$

Note that the number of boarding passengers depends on the time gap between arrival time $t^a_{j,m}$ of the current bus, the arrival time of the last vehicle $t^a_{j-1,m}$ visiting stop m, and the remaining passengers at stop m. This is equivalent to an assumption that no passengers arrive during the dwelling process and is similar to many other studies in the literature (Daganzo 2009; Cats et al. 2010). This assumption can be relaxed by incorporating the boarding process recently introduced in Wu, Liu, and Jin (2017). After the boarding process of vehicle j at stop m, the number of left-over passengers in the residual queue that cannot board the bus j can be updated as follows:

$$l_m(t^a_{j,m}) = l_m(t^a_{j,m}) - B_{j,m}. \tag{11}$$

The occupancy of bus j leaving stop m then can be updated after the departure time $t^d_{j,m}$:

$$Occ_{j,m} = Occ_j(m-1) + B_{j,m} - A_{j,m}. \tag{12}$$

Finally, the dynamical equation of the proposed model can be rewritten as

$$t^d_{j,m} = t^d_{j,m-1} + \frac{L_m}{v_j(m-1)} + D_{j,m}. \tag{13}$$

Equation (13) defines the departure time of bus j from stop m as a function of the departure time from the stop m−1, plus the travel time between stops m−1 and m and the dwell time at stop m.

To simplify the modelling process, many of the existing bus route simulation studies assume that no overtaking occurs in the system, for a detailed discussion see Wu, Liu, and Jin (2017). However, without the overtaking behaviour it would not be possible to account for leap-frog bunching. We focus on systems where overtaking is allowed. The vehicles are allowed to freely overtake each other in the proposed schedule-following model. As a vehicle j overtakes its previously leading vehicle j−1, we simply swap the indexes of these two vehicles. This essentially means that the two vehicles swap their schedule. Recall in Equation (3) that their speeds only depend on the schedule. Vehicle index swapping has been made possible due to the fact that the passenger arrival process (Algorithm 1) is independent of the vehicle arrivals, and $l_m(t)$ only depends on the arrival time of any bus at stop j.

The next section provides numerical examples and evaluates the performance of the proposed model.

5. Model performance

To demonstrate the performance and characteristics of the proposed schedule-following model, we perform some rigorous numerical sensitivity analysis and compare it to the existing bus-following (Nagatani 2001; Hill 2003) and traffic-following (Toledo et al. 2010; Cats et al. 2010) models.

5.1. Bus-following model formulation

The bus-following model is governed by a dynamical equation as described in Hill (2003):

$$t_{j,m} = t_{j,m-1} + \lambda\gamma\delta t_{j,m-1} + \frac{L}{V_{j,m-1}}, \tag{14}$$

where $t_{j,m}$ is *both* the arrival and departure time of bus j at stop m; $\delta t_{j,m} = t_{j,m} - t_{j+1,m}$ is the time headway. Note that unlike the proposed schedule-following model, there is no separation of the departure time and arrival time at a bus stop. γ is the time it takes for each passenger to board. $\lambda\gamma\delta t_{j,m-1}$ is the estimation of dwell time. The mean speed $V_{j,m-1}$ is calculated using the following velocity function:

$$V(\delta x_j) = v_{\min} + (v_{\max} - v_{\min}) \times \frac{\tanh[\varphi(\delta t - t_c)] + \tanh(\varphi t_c)}{1 + \tanh(\varphi t_c)}, \tag{15}$$

where t_c is the safe (critical) headway between two buses. φ acts as a spread-out parameter for the hyperbolic tangent factor.

5.2. Traffic-following model formulation

The traffic-following model uses a similar dynamical equation to Equation (13) (Toledo et al. 2010). Its mean speed is calculated using the following function:

$$f(n) = \begin{cases} v_{\max} & k < k_{\min}, \\ v_{\min} + (v_{\max} - v_{\min}) \times \left[1 - \left(\dfrac{k - k_{\min}}{k_{\max} - k_{\min}}\right)^a\right]^b & k \in [k_{\min}, k_{\max}], \\ v_{\min} & k > k_{\max}, \end{cases} \tag{16}$$

where k is the link density, $k_{\min}$ and $k_{\max}$ are the minimum and maximum density thresholds, a and b are parameters. The traffic-following model (Toledo et al. 2010) formulates the dwell time $D_{j,m}$ as

$$D_{j,m} = \beta_1 + \max(PT_{j,m}^{\text{front}}, PT_{j,m}^{\text{rear}}) + \beta_2 \times \delta_j^{\text{bay}} + \beta_3 \times \delta_{j,m}^{\text{full}} + \nu_{j,m}, \tag{17}$$

where $PT_{j,m}^{\text{front}}$ and $PT_{j,m}^{\text{rear}}$ are the total passenger service time at the front and rear doors, respectively. δ_j^{bay} equals 1 if the bus stop is a bay, and 0 otherwise. $\delta_{j,m}^{\text{full}}$ equals 1 if the stop is completely occupied and 0 otherwise. β_1 is the dead time required for door closing and opening; β_2 is the delay due to approaching a bay stop; β_3 is the delay due to approaching a fully occupied stop and $\nu_{j,m}$ is an error term. The passenger service times $PT_{j,m}^{\text{front}}$ and $PT_{j,m}^{\text{rear}}$ can be calculated as

$$PT_{j,m}^{\text{front}} = \alpha_1 \times p_{\text{front}} \times A_{j,m} + \alpha_2 \times B_{j,m} + \alpha_3 \times \delta_{j,m}^{\text{crowded}} \times B_{j,m}, \tag{18}$$

$$PT_{j,m}^{\text{rear}} = \alpha_4 \times (1 - p_{\text{front}}) \times A_{j,m}, \tag{19}$$

where p_{front} is the fraction of passengers that alight from the front door. α_1 and α_4 are the time for each passenger to alight from the front and rear doors, respectively. α_2 is the time for each passenger to board a uncrowded bus, and α_3 is the additional time for each passenger to board a crowded bus. $\delta_{j,m}^{\text{crowded}}$ equals 1 if the bus exceeds the number of seats C_s, and 0 otherwise. The number of boarding passengers $B_{j,m}$ is estimated by a Poisson process, while the number of alighting passengers $A_{j,m}$ is estimated by Equation (9).

5.3. Parameter settings

Table 1 shows the parameter settings used in the numerical sensitivity analysis for the three models. The parameters for the time required for boarding, alighting and door closing are adopted from Bertini and El-Geneidy (2004), which equal 3.6, 0.85 and 5.8 respectively. The global network setting parameters are: $L = 0.5$ km, $M = 20$, $N = 6$, $\omega = 0.3$ and $H = 5$ min for all models.

Note that for our comparison study we use the same passenger arrival process as in Algorithm 1 for estimating the number of arrived passengers $I_m(t)$ and boarding passengers $B_{j,m}$ for both the schedule-following and traffic-following models. The reasons for this are first to enable a consistent and fair comparison, and second, to enable vehicle overtaking in both the proposed schedule-following and traffic-following models.

For both the schedule-following and traffic-following models, we assume that the probability of alighting at each stop ρ_m is fixed within the same time period. Parameter ρ_m linearly increases from 0 $\rightarrow$ 1 as m increases from 1 $\rightarrow$ M, so that passengers are more likely to alight downstream of the bus route, and all on-board passengers would alight at the last stop M. The bus-following model ignores the effect of alighting passengers, but uses the same arrival rate λ as the other two models. Table 1 shows the parameters used in each model under comparison.

The following sections evaluate the three bus route models in two scenarios where (1) there is no perturbation introduced (nothing disrupts the buses as they travel between stops and the passenger arrival rate is deterministic) and (2) random perturbations are introduced at every link.

Table 1. Parameter settings for the three bus route models.

Schedule-following	Bus-following	TF	TF (cont'd)
$v_{\text{max}} = 60$ km/h	$v_{\text{max}} = 60$ km/h	$v_{\text{max}} = 60$ km/h	$\beta_3 = 0$
$v_{\text{min}} = 40$ km/h	$v_{\text{min}} = 40$ km/h	$v_{\text{min}} = 40$ km/h	$\delta_j^{\text{bay}} = 0$
$\theta_1 = 5.8$ s	$\varphi = 1$	$k = 0.5$	$\delta_{j,m}^{\text{full}} = 0$
$\theta_2 = 3.6$ s	$t_c = 1$ min	$k_{\text{max}} = 1$	$\nu_{j,m} = 0$
$\theta_3 = 0.85$ s	$\gamma = 3.6$ s	$k_{\text{min}} = 0$	$\alpha_1 = 0.85$ s
$\lambda = 1$ pass/min	$\lambda = 1$ pass/min	$a = 1$	$\alpha_2 = 3.6$ s
$C = 80$ passengers		$b = 1$	$\alpha_3 = 3.6$ s
		$\beta_1 = 5.8$ s	$\delta_{j,m}^{\text{crowded}} \in [0, 1]$
		$\beta_2 = 0$	$p_{\text{front}} = 0.5$
		$\lambda = 1$ pass/min	$C_s = 40$ passengers

TF, traffic-following model.

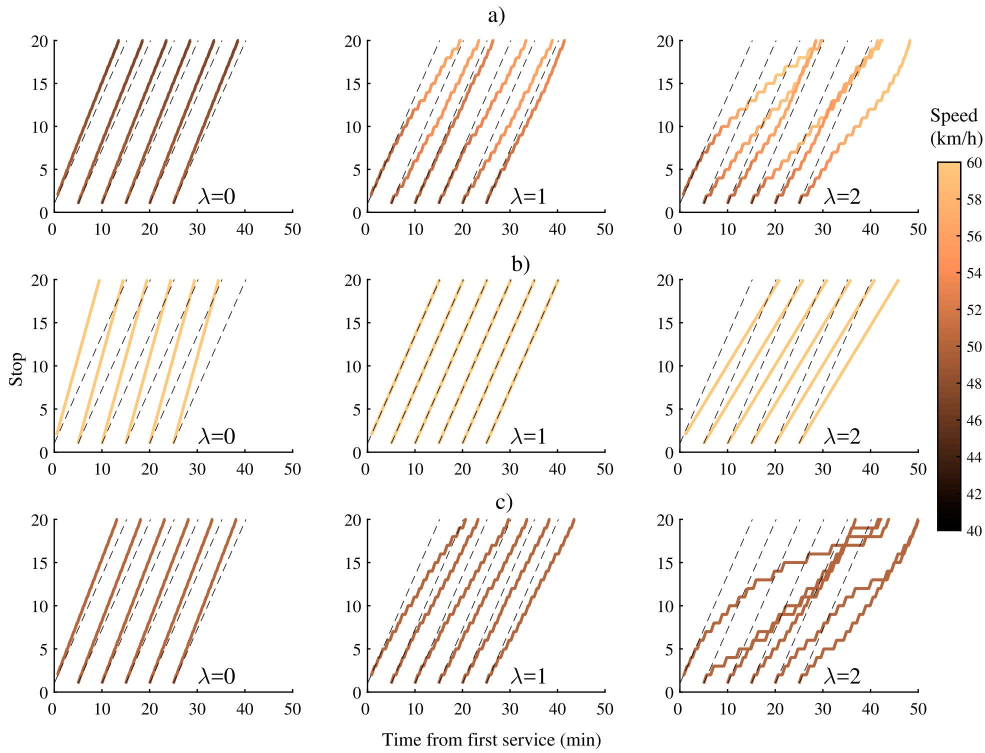

Figure 4. Sensitivity of λ: (a) schedule-following model, (b) bus-following model and (c) traffic-following model.

5.4. Study cases

5.4.1. Scenario 1: no perturbation

This section compares the three models being studied without adding any random perturbations to their operation. This demonstrates the operational characteristic of the models. Figure 4 shows the impact of λ (the number of passengers per minute) on bus trajectories from the (a) schedule-following model, (b) bus-following model and (c) traffic-following model. The bold lines are trajectories of buses, where a darker colour means a lower mean speed. The dashed lines are the scheduled arrival times at stops.

All three models capture the fact that buses experience more delays as λ increases. The following are the two main differences between the models:

- In the proposed schedule-following model, drivers try to adhere to the schedule by adapting their speeds, while drivers in the other models do not consider a predefined schedule. Buses in the schedule-following model speed up when behind schedule and slow down otherwise, while buses in bus-following and traffic-following models maintain constant speeds regardless of the on-time performance. The schedule-following behaviour is consistent with the observation in Figure 1.
- The leapfrogging phenomenon at larger values of λ is captured in the proposed schedule-following and traffic-following models as a result of the Poisson-based stochastic passenger arrival process. While the modelling results at $\lambda = 1$ passenger/s are similar for the schedule-following and traffic-following models, the latter shows significantly more bunching at $\lambda = 2$ passenger/s than the former. This is because the proposed schedule-following model has a speed adaptation feature where drivers try to speed up to adhere to schedules, while the speed in the traffic-following model depends on the value of traffic density k. On the other hand, a larger passenger arrival rate does not create disturbances in the bus-following model because the deterministic passenger arrival process delays every bus equally.

Figure 5 demonstrates the speed adaptation feature of the proposed model (Figure 5a) compared to the bus-following model (Figure 5b) and traffic-following model (Figure 5c). On the left side of Figure 5(a and b), buses are constrained by a narrow speed range $[v_{\min}, v_{\max} = [45, 55]$ km/h of the traffic, whereas on the right side, the constraint is very large. Traffic-following is a special case, where the bus speed is calculated using the link density k. Therefore, the speed range $[v_{\min}, v_{\max}]$ on both sides of Figure 5(c) is [10,90], but the left side has high traffic density ($k = 0.5$) while the right side has no traffic density ($k = 0$). In practice, we can say that the right side of Figure 5 simulates a system with dedicated rights of way, such as busways or bus lanes, where buses do not interact with the surrounding traffic. The left side of Figure 5 on the other hand represents a system with shared rights-of-way. λ equals 1.5 passenger/s in this experiment.

In the left side of Figure 5, the narrow range of possible speeds causes limited speed variation between the three models. All three models show a similar colour in their trajectories. There are significant differences in vehicle speeds on the right side of Figure 5 because in those experiments bus speeds can vary between 10 and 90 km/h. Buses in the bus-following (Figure 5b) and traffic-following (Figure 5c) models do not change their operating speed. They almost always travel at the maximum speed. Whereas buses in the schedule-following model adapt their speed to adhere to the schedule. By comparing the two sides of Figure 5(a), we notice that the overall performance of the bus route is better if there is the freedom to adapt bus speeds for schedule adherence. This is similar to practice, where buses on dedicated rights-of-way are better able to keep to their schedules than buses on shared roads due to being unconstrained by the surrounding traffic (Chen, Yushi Zhang, and Guo 2009).

Another way to evaluate the two models is to use a phase diagram of system states. States here refer to the headway between vehicles: whether the headway is uniform; unstable; or if buses are bunched. Simulations are executed with different values of λ and H using the three models, and repeated 10 times to reduce numerical instability. A 4-region Phase diagram is illustrated in Figure 6. The definitions of these phases follow Luo et al. (2012).

- Phase region I: The uniform state, where all simulated headways are more than half of the scheduled headway. Formally, $\forall j, m : \delta t_{j,m} > H/2$ with $j \in [1 \ldots N]$ and $m \in [1 \ldots M]$, where H is the scheduled headway. In this phase, buses maintain a regular headway. Schedule-following and traffic-following models exist in this phase when the demand is low, while the bus-following model always stays in this phase.
- Phase region II: The lack of capacity state, where the simulated headways are still more than $H/2$, but some buses reach full capacity. Formally, $\forall j, m : \delta t_{j,m} > H/2$ but $\exists j, m : Occ_{j,m} == C$. When both λ and H are high, most of the buses will reach their capacity. While the headway between them is relatively uniform because they share the same workload, there will be passengers who cannot board

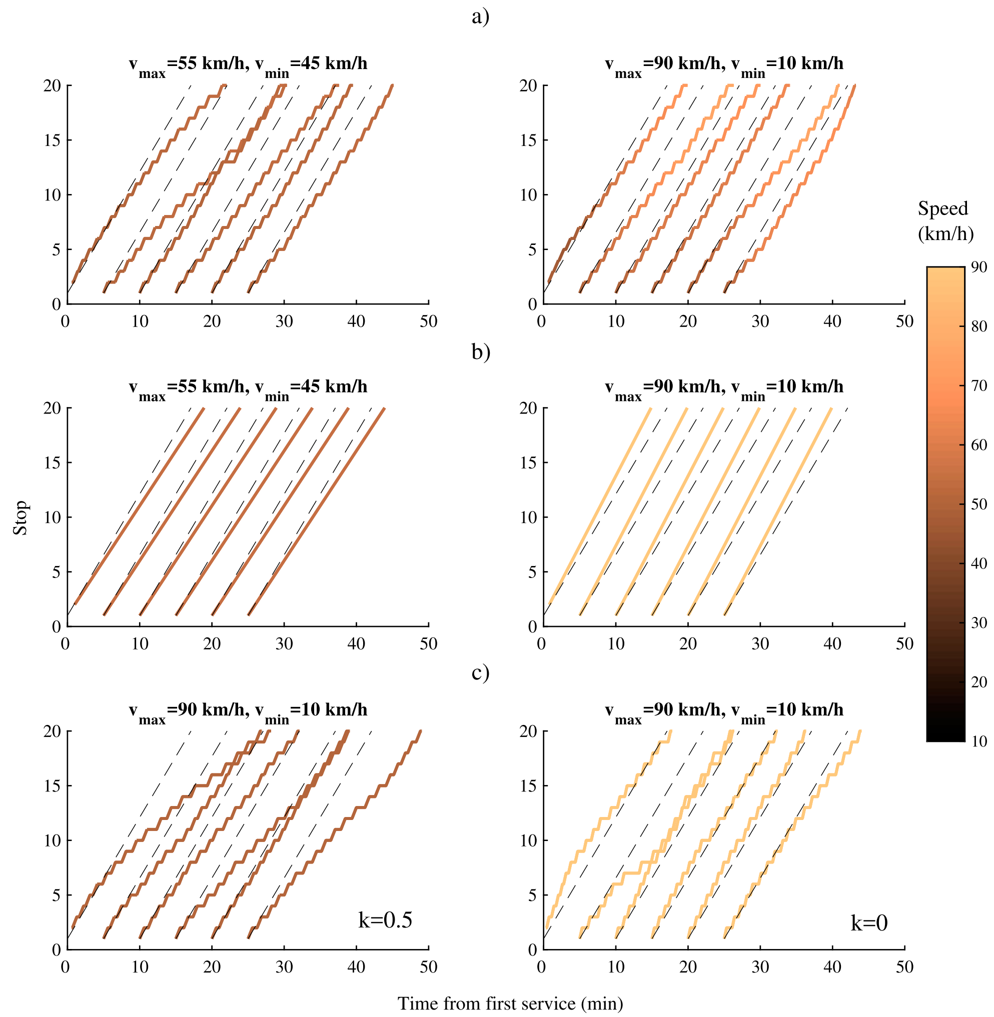

Figure 5. Impact of speed constraints on on-time performance: (a) schedule-following model, (b) bus-following model and (c) traffic-following model.

a bus. This region appears only in the proposed schedule-following and traffic-following models thanks to the existence of vehicle capacity.

- Phase region III: The unstable state, where at least one simulated headway $\delta t_{j,m}$ drops below half of the scheduled headway, but more than the critical headway t_c. Formally, $\exists j, m : H/2 > \delta t_{j,m}$ and $\forall j, m : \delta t_{j,m} > t_c$. This region appears in the proposed schedule-following model with a V-shape around region IV. The traffic-following model also enters this phase occasionally, but Phase region IV is a lot more common.
- Phase region IV: The congested state, where bus-bunching occurs. Formally, $\exists j, m : \delta t_{j,m} < t_c$. This region only appears in the proposed schedule-following and traffic-following models.

The bus-following model always yields the uniform state in this experiment, due to the model's deterministic nature. Even at high demand and frequency, buses maintain the same headway between each other, which results in the system always being in Phase region I.

The four phase regions are consistent with Luo et al. (2012) and the observations using real data in Liu and Sinha (2007). At low demand, both schedule-following and traffic-following models are stable, and they become more unstable as λ increases. The traffic-following model appears to be quite

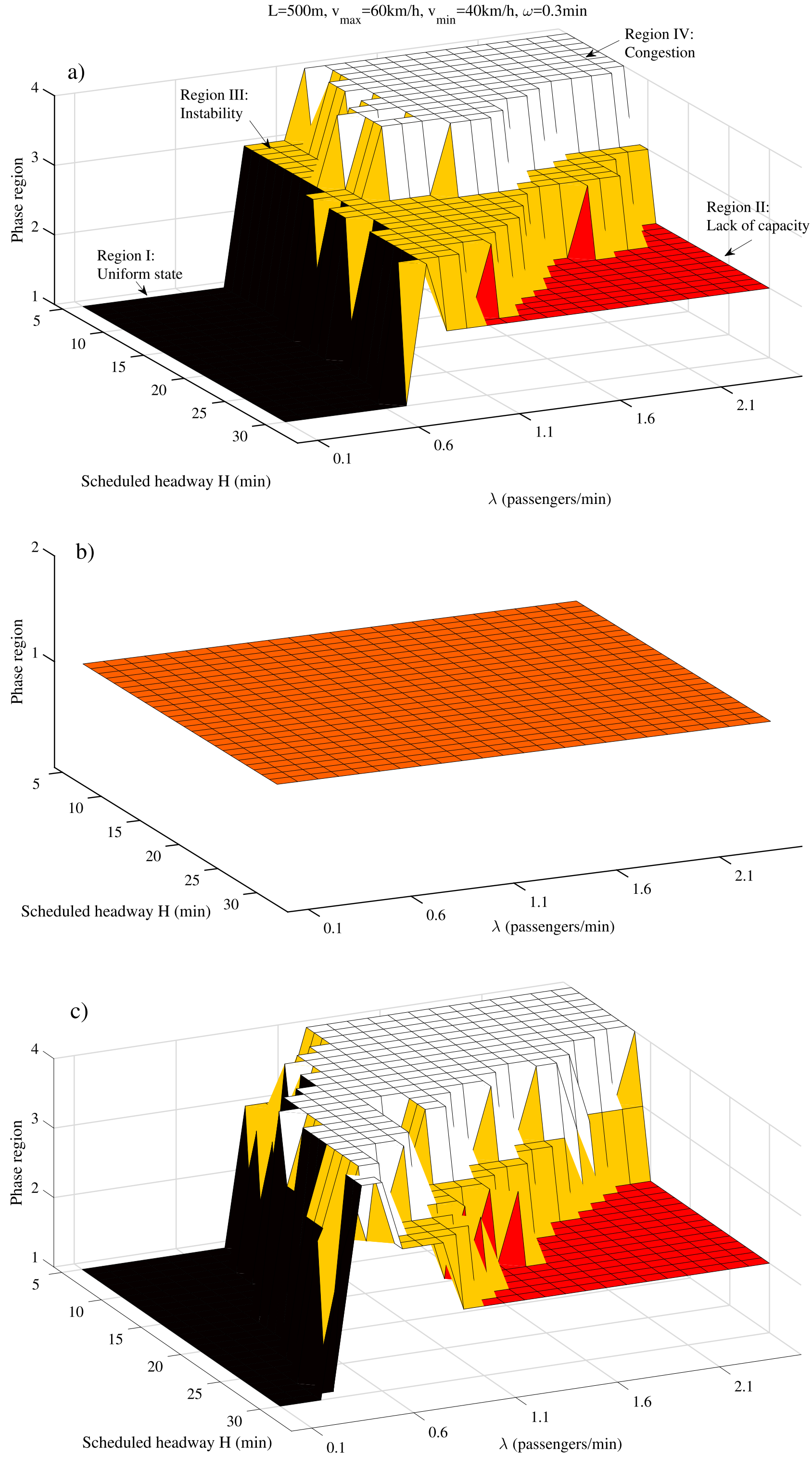

Figure 6. Phase diagram analysis of (a) schedule-following model, (b) bus-following model and (c) traffic-following model.

random, because Phase IV also appears when the scheduled headway H is large and the passenger demand λ is low. Using the same system of randomness as the traffic-following model (a Poisson process), the performance of the proposed schedule-following model seems to be more resilient due to the speed adaptation feature.

5.4.2. Scenario 2: random perturbation at every link

This scenario evaluates the stability of bus system when the travel time between stops includes a random perturbation. The purpose of this experiment is to evaluate the stability of the three bus route models when the observed data contain noise, or when the system is exposed to random disturbances in operation such as random travel time between links that might be caused, for example, by the surrounding traffic. The dynamical equations of the proposed schedule-following and traffic-following models now read:

$$t^d_{j,m} = t^d_{j,m-1} + \frac{L_m}{v_j(m-1)} + D_{j,m} + \xi \times r_j \tag{20}$$

and similarly for the bus-following model:

$$t^d_{j,m} = t^d_{j,m-1} + \lambda\gamma\delta t_{j,m-1} + \frac{L}{V_{j,m-1}} + \xi \times r_j \tag{21}$$

where r_j is a uniformly distributed random number between $[-1, 1]$ and ξ is the perturbation magnitude. $\xi = 0$ means no perturbation, while $\xi = 1$ means that the deviation from the scheduled dispatch time is $[-1, 1]$ minute. Figure 7 shows the bus trajectories from the three models when $\xi = 0.1$. This perturbation setting is consistent with Hill (2003).

As $\xi = 0.1$, the deviation is only between $[-6, 6]$ seconds at each stop, which is hardly noticeable in practice. It is expected that the models should show relatively similar simulated trajectories as in Figure 4. However, only the proposed schedule-following model (Figure 7a) exhibits unnoticeable differences to the bus trajectories. This is because, as with real practice, drivers can easily recover from small deviations by adapting their speeds. Conversely, the bus-following (Figure 7b) and traffic-following models (Figure 7c) show much less realistic trajectories under noise, where the small perturbations evolve into significant service disturbances especially at large λ. Figure 7(c) shows that bus bunching now appears even at low passenger demand ($\lambda = 1$). These results are further investigated in the phase diagram in Figure 8.

The proposed schedule-following model shows a very similar phase diagram to that produced in the first scenario (Figure 6), with slightly more stochasticity due to the random perturbation. However, major changes in the phase diagram can be found in the bus-following model, where the unstable (Phase III) and congested state (Phase IV) can now be found. The traffic-following model shows even more stochastic results than before, as the Phase I (stable) and Phase IV (congested) are occasionally mixed up when $\lambda \approx 0.6$ passengers/min.

5.5. Model performance: discussion

Two scenarios have been developed to evaluate and compare three bus route simulation models: bus-following, traffic-following and proposed schedule-following. Figures 4–8 illustrated the results. The two scenarios showed that the proposed model captures the three important bus operation dynamics:

- Buses adhere to schedules when the demand is low, but they cannot do so when the demand is large. When bus speed is not constrained by traffic, better schedule adherence can be obtained.
- A late bus may cause bus bunching when the following bus catches up with it.
- The leapfrogging phenomenon occurs as two or more buses cannot separate from each other.

There are two main differences between the bus-following model and the proposed schedule-following model. First, the bus-following model is deterministic, meaning that it is always in the stable

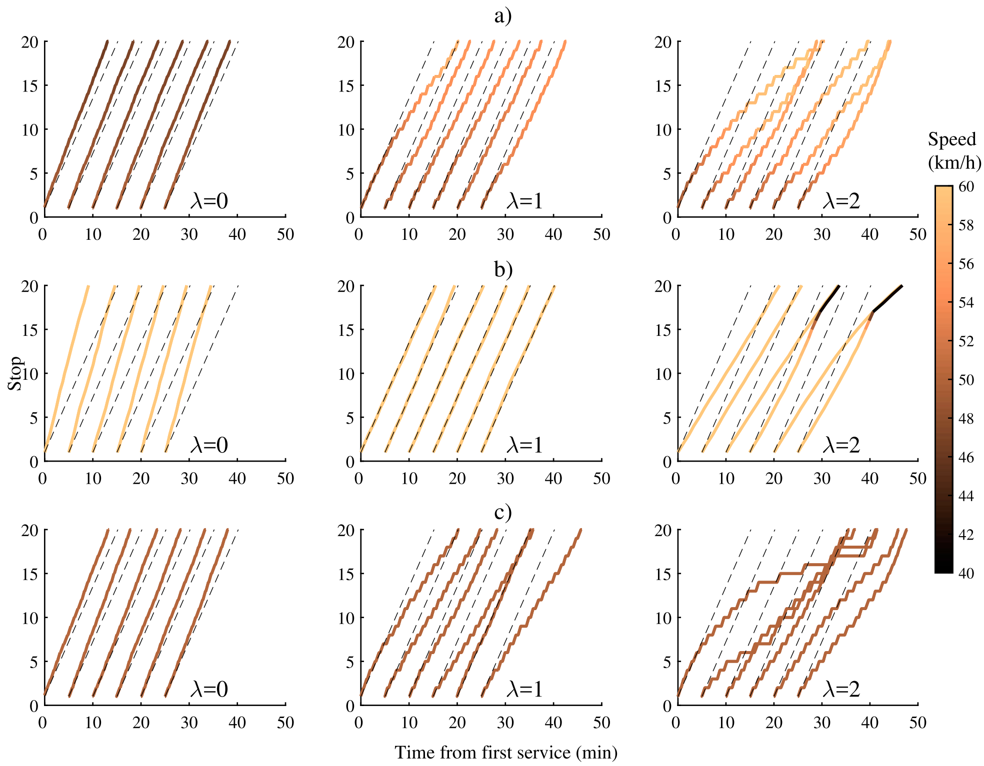

Figure 7. Bus trajectories after random perturbations: (a) schedule-following model, (b) bus-following model and (c) traffic-following model.

phase (Phase I, Figure 6), until some random noise is introduced (Figure 8). Conversely, the proposed schedule-following model is stochastic and is able to capture four phases of a bus operation with or without random noise in the system. Second, the bus-following model assumes that a bus would slow down when approaching another bus, similar to a collision-free car-following model. This means that the bus bunching phenomenon, which is a common occurrence in real systems, will not occur in the standard bus-following model. On the other hand, the proposed schedule-following model captures both bus bunching and leapfrogging phenomena.

Compared to the traffic-following model, the proposed schedule-following model shows two major differences. First, the traffic-following model aims to model buses as a component of the traffic system. Cats et al. (2010) use a traffic simulation model as the traffic environment, and Toledo et al. (2010) use traffic density data to model the traffic state on the links where buses are operating. However, this explicit approach requires a substantial amount of traffic data or a comprehensive traffic model (Cats et al. 2010) to account for the surrounding traffic. It is very challenging to collect traffic data for every bus link in practice because data are often only available at major road links, while buses tend to also cover minor links to serve residential areas. The proposed schedule-following model, on the other hand, can model the bus speed under the influence of traffic conditions through the parameters $v_{\max}$ and $v_{\min}$. This is of course a vast simplification of the broader traffic patterns, but it provides a simple way to calibrate the model without traffic data. Second, the proposed schedule-following model is a better candidate for simulating a conventional schedule-based bus system. Buses in the schedule-following model try to maintain a predefined schedule, similar to bus drivers in practice. This is best shown in Figure 7, where buses in the schedule-following model can recover a schedule at low demand, and buses in the traffic-following model become bunched even with low demand after a minor perturbation.

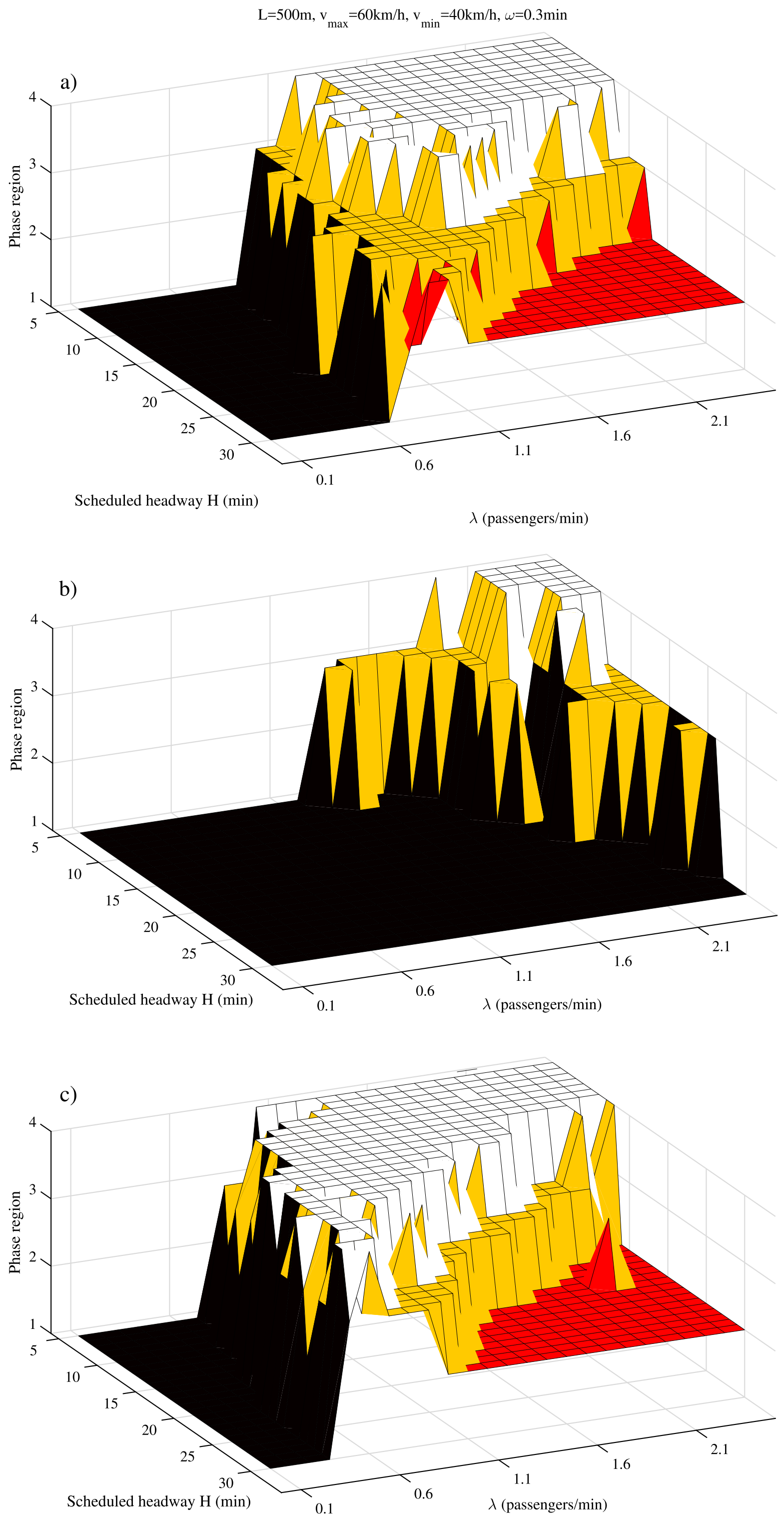

Figure 8. Phase diagram analysis of (a) schedule-following model, (b) bus-following model and (c) traffic-following model under small perturbation.

It is clear that the proposed schedule-following model captures well the dynamics in a conventional schedule-based bus system. We will show in the next section how it is calibrated to the observed data.

6. Model calibration

This section calibrates the proposed schedule-following model against the observed Automatic Vehicle Location (AVL) and Smart Card data from Route 555 in Brisbane, Australia. It aims to demonstrate that the proposed model is capable of reproducing reality as seen from observations. The schedule-following model is run multiple times and the distribution of simulated vehicle headway between buses is collected. These are then compared to the observed headway from the AVL data. Recall that Route 555 operates on a busway, so there is no impact from traffic. We only calibrate the proposed schedule-following model, not the traffic-following and bus-following models, because there are no traffic data that can be used to calibrate the traffic-following model, and because the deterministic bus-following model has no variation in vehicle headway to get a distribution.

6.1. Data description

This paper uses 4 months of AVL and Smart Card data from July to October 2013. Each AVL record includes information about each visit to a bus stop, including route number, trip ID, vehicle ID, scheduled departure time, observed arrival time and observed departure time. The time headway between vehicles is estimated as the difference between the departure time from the same stop of two adjacent buses. In addition to the AVL data that provides the arrival and departure times, the calibration also uses the Smart Card data for the same route, giving the number of boarding and alighting passengers at each stop. Each Smart Card record includes boarding and alighting locations, time stamps as well as a hashed unique ID of the smart card used for the journey. Only working days are used for calibration. The study period is 7:15 AM to 9:15 AM on the inbound direction, thus N equals 9, H equals 15 minutes and M equals 12 stops.

6.2. Calibration problem formulation

Parameter calibration is an optimisation problem to minimise some error index $PI(\pi)$ over all $\pi \in \mathbb{R}^k$. A solution $\pi = (\pi_1, \pi_2, \ldots, \pi_k)$ refers to a set of model parameters and k denotes the number of parameters in this set. Let π_* denote the optimal set of parameters, that is:

$$\pi_* = \text{argmin} \quad PI(\pi), \quad \pi \in \mathbb{R}^n. \tag{22}$$

Equation (22) is equivalent to finding π_* such that $PI(\pi_*) \leq PI(\pi) \quad \forall X \in \Pi$, where Π is a constrained parameter space such that $\Pi \in \mathbb{R}^k$. The error index $PI(\pi)$ is the difference between model outputs and observed data.

The challenges in this calibration problem come from the fact that the schedule-following model is stochastic, i.e. the same solution π may yield different realisation $PI(\pi)$. To reduce this stochastic effect, we evaluate each solution π a hundred times (replications) and compare the distribution of outputs with the distribution of the observed data. Solving this optimisation problem by hand is tedious, so we propose the use of a population-based Monte Carlo learning algorithm, based on the Cross-Entropy Method (CEM) (Rubinstein 1999) for optimising the parameters of the model.

CEM originated from the field of rare event simulation, where even small probabilities need to be estimated. It has been developed into a combinatorial multi-extremal optimisation (Rubinstein 1999). Formally, CEM maintains and develops a *probability distribution* over a generation of solutions for an optimisation problem (model parameters in this case). At each iteration, new solutions are drawn from this distribution and evaluated. After ranking the solutions according to a predefined *performance index*, the best ones are selected to develop an improved probability distribution of the parameters, which will then be used to create a new generation of solutions for the next iteration, until certain criteria are met (a.k.a. *convergence*). CEM has been chosen over other popular optimisation methods such as Genetic Algorithms (Heppenstall, Evans, and Birkin. 2007), because its probabilistic nature facilitates the calibration of stochastic models (Ngoduy and Maher 2012). Interested readers may refer to Rubinstein (1999) for a more detailed account of the CEM. Pseudo-code for CEM for the Normal distribution is also included in Appendix A of this paper.

The proposed schedule-following model is driven by five parameters: $[v_{\min}, v_{\max}, \theta_1, \theta_2, \theta_3]$. These five parameters will be calibrated in this section. Parameter λ is estimated from the Smart Card data by taking the mean number of boarding passengers per minute over the studied period. Other parameters L, ω, M, H are taken directly from the AVL data. The objective function is formulated as

$$\min\ z = E\left[\frac{1}{M}\sum_{m=1}^{M}\sqrt{\sum_{h=0}^{2H}(P(h_{\pi,m} = h_m) - P(\tilde{h_m} = h_m))^2}\right] \tag{23}$$

subject to:

$$\pi_j^{\max} \geq \pi_j \geq \pi_j^{\min}, \tag{24}$$

where h_π and $\tilde{h}$ are the time headway obtained from the proposed schedule-following model and observed AVL data, respectively. $P(h_{\pi,m} = h_m)$ and $P(\tilde{h_m} = h_m)$ are the probabilities that the simulated or actual headway are equal to a value h at stop m. These values range from 0 (bus-bunching) to $2H$ minutes. By this definition, $z \in [0, 1]$ and $z \to 0$ represent the better fit whereas $z \to 1$ indicates the worse fit. Each set of solutions contains five values for $[v_{\min}, v_{\max}, \theta_1, \theta_2, \theta_3]$, where the calibration is subjected to the predefined upper and lower bounds $[\pi_j^{max}, \pi_j^{min}]$ of each parameter. We replicate each solution 100 times to build up a comparable sample size of h_π to compare with the observed data h. After several empirical tests, we adopt the following hyper-parameters for the CEM:

- Samples: 1000 solutions,
- Elite samples ratio: 20%.

6.3. Calibration results

The calibration process is considered 'converged' if the mean and standard deviation of z over 1000 samples satisfy the following two criteria:

- After five iterations, the mean of z (over 1000 samples) do not reduce by more than 5%.
- The standard deviation of z (over 1000 samples) is close to zero.

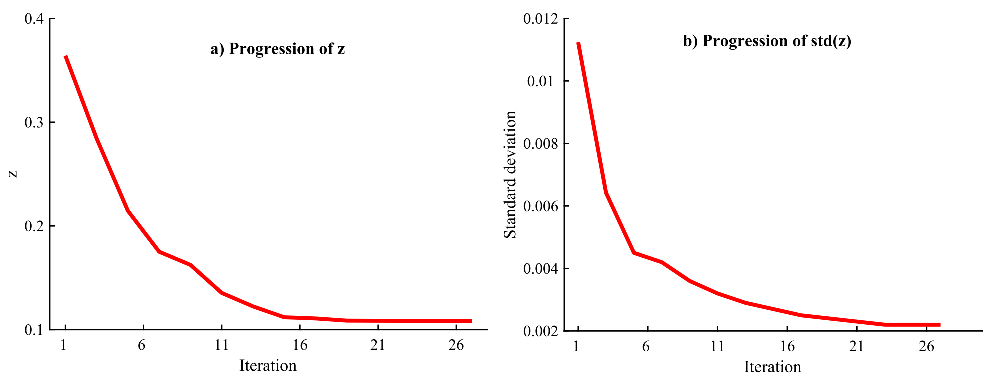

Figure 9. Progression of the performance index z (over 1000 samples): (a) convergence of the mean of z and (b) convergence of the standard deviation of z.

Table 2. Schedule-following model parameters and performance index after calibration.

Calibrated parameters	Value
v_{min}	17.06 km/h
v_{max}	51.83 km/h
θ_1	23.4 s
θ_2	3.9 s
θ_3	1.54 s
Best z	0.057

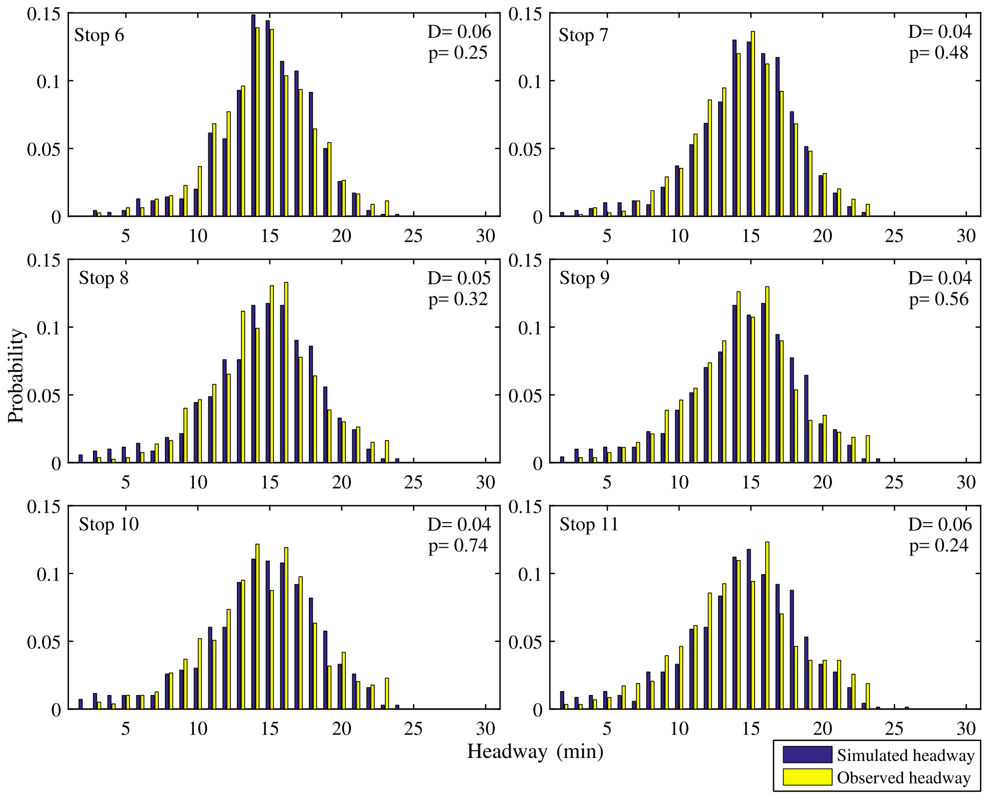

Figure 10. Headway distribution at some stops along the Route 555.

Figure 9 shows the progression of z after 27 iterations.

Figure 9 shows that a reasonable convergence has been reached where the standard deviation of the objective function approaches zero and the value of the expected objective function does not improve anymore. Table 2 shows the best parameters settings and performance index z of the proposed schedule-following model after calibration.

Figure 10 compares the simulated headway h_π of the calibrated model and the observed headway h from AVL data at some stops along the Route 555.

The distribution of h_π and h is very similar. Two-sample Kolmogorov–Smirnov tests are also conducted to compare the two distributions. The results are also presented in Figure 10, where D is the Kolmogorov–Smirnov statistic or the absolute max distance (supremum) between the CDFs of the two samples. p is the p-value of the Kolmogorov–Smirnov test. The null hypothesis that h_π and h come from the same distribution can only be rejected at the 95% confidence level if the p-value is less than 0.05.

Figure 10 therefore shows that the proposed model can reproduce a similar headway to that exhibited by the observed data.

7. Conclusion

This paper develops a new innovative bus route simulation model to capture the dynamics of buses as observed in AVL data. The model captures three important phenomenon: (1) buses follow the schedule and aim to adhere to the schedule as closely as possible, (2) bus bunching occurs when the following bus catches up with a late bus, especially at times of high demand, and (3) leapfrogging occurs when two or more buses cannot separate from each other.

When evaluating the numerical simulation results using time-space and phase diagrams, the proposed model shows the most realistic dynamics compared to two popular types of bus simulation models: the bus-following model and the traffic-following model. Buses in the proposed model adjust their cruise speed to adhere to the schedules, which is similar to the practice when bus drivers have to follow a predefined schedule. The proposed schedule-following model also shows all four operational phases, similar to the empirical findings in Liu and Sinha (2007) using only five governing parameters.

The model is calibrated using the observed AVL and Smart Card data. The case study demonstrates that the proposed model reproduces similar headway to the observed data. Further developments include incorporating schedule-following and traffic-following mechanisms and extending the model to system-wide networks to augment the model's applicability to policy makers in practice. The model can also be used to investigate the causes and impacts of bus bunching and leap-frog bunching.

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Appendix. Cross-entropy method for optimisation

This Appendix describes the pseudocode for the cross-entropy method for normal distribution (Rubinstein 1999).

Algorithm 2: Cross-entropy method for normal distribution

Set $p = (\mu_1, \sigma_1, \mu_2, \sigma_2, ..., \mu_K, \sigma_K)$ %Initial distribution parameters
Set M %Number of stops
Set T % Maximum iteration number
Set I % Maximum iteration number
Set ρ % Set selection ratio
for *t from 1 to T* **do**
  %Main CEM loop
  for *i from 1 to I* **do**
    Draw $y^{(i)}$ from $\mathcal{N}(\mu, \sigma)$ %Draw I samples
    Compute $f^i := f(y^{(i)})$
  end
  Sort f^i-values %Order by decreasing magnitude
  $\gamma \leftarrow f_{\rho.I}$ %Set threshold
  $L_\gamma \leftarrow \{y^{(i)} | f(y^{(i)}) \leq \gamma$ %Collect elite samples
  $\mu'_j = \frac{1}{L_\gamma} \sum_{i=1}^{L_\gamma} \mu_{i,j}$ %Update μ
  $\sigma'_j = \frac{1}{L_\gamma} \sum_{i=1}^{L_\gamma} \sigma_{i,j}$ %Update σ
  $\mu_j \leftarrow \alpha \mu'_j + (1 - \alpha)\mu_j$ %Update with step size α
  $\sigma_j \leftarrow \alpha \sigma'_j + (1 - \alpha)\sigma_j$ %Update with step size α
end