Optimising an Agent-Based Model to Explore the Behaviour of Simulated Burglars

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Abstract Agent-based methods are one approach for modelling complex social systems but one issue with these models is the large number of parameters that require estimation. This chapter examines the effect of using a genetic algorithm (GA) for the parameter estimation of an agent-based model (ABM) of burglary. One of the main issues encountered in the implementation was the computation time required to run the algorithm. Nevertheless a set of preliminary results were obtained, which indicated that visibility is the most important parameter in the decision of whether to burgle a house while accessibility was the least important. Such tools may eventually provide the means to gain a greater understanding of the factors that determine criminological behaviour.

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1 Introduction

Social systems are incredibly complex due to the large number of interacting elements and many underlying processes that are simply not understood. Moreover, these processes are generally non-linear such that small changes in system parameters can have large effects on the outcomes of the system as a whole. Complex systems are also characterised by self-organisation whereby spontaneous behaviours emerge through the interactions of the individuals and the feedbacks in the system, e.g. the flocking behaviour of birds or the movements of financial markets (Cilliers, 1998). Agent-based models (ABMs) have been developed as one technique for modelling complex systems where the individuals or 'agents' of the system are explicitly represented in these models. Agents are independent entities that are capable of interacting with each other and with their environment. The agents make assessments of their situation over time (or during each iteration of the model) and then make decisions in response to these assessments (Bonabeau, 2002). By providing realistic environments and rules that are based on observed and expected patterns of human behaviour, it is possible to create models that can simulate real world systems (Moss and Edmonds, 2005).

Classic examples of ABMs are the Sugarscape model of Epstein and Axtell (1996), which simulates wealth accumulation through sugar harvesting in a simple environment, and Schelling's (1971) model of segregation, which has been simulated by a number of researchers in the past using an ABM approach (see e.g. Omer, 2005 and Crooks, 2008). ABMs are now being applied in a variety of different domains, e.g. ecology (Grimm and Railsback, 2005), economics (Tesfatsion and Judd, 2006) and more recently, criminology (Malleson 2009; 2010a, b, c).

Although ABMs represent a way to capture complexity in social systems, they have issues related to parsimony, i.e. they contain a potentially large number of parameters. Some parameters can be determined through expert knowledge or can be derived from field measurements or social surveys. However, many others are unknown and therefore require a method to determine their values. The need to calibrate a model is not limited to ABMs and many different methods of search and optimisation are available. However, classical search methods are not effective in finding large numbers of parameters so other methods such as genetic algorithms (GAs) are needed.

Despite the fact that GAs are well suited to high dimensional parameter estimation, there are not many examples of the use of GAs in the development of geospatial ABMs. GAs have been used to calibrate cellular automata models of urban land use (e.g. Goldstein, 2004; Li et al., 2007; Shan et al., 2008), which might be considered as pre-cursors to ABMs but which are still used today for studying urban form and land use change. One of the most notable examples is the work by Heppenstall et al. (2007), who used a GA to calibrate the parameters of an agent-based retail petrol market model. In the model, the market petrol retailers, i.e. the petrol stations, compete for customers within localised, overlapping areas and were therefore represented by agents. Knowledge was embedded in each agent regarding the initial starting price, production costs, and the prices of those stations within their immediate neighbourhood. A series of rules were then applied to each agent in order to effect petrol price adjustments. A GA was used to optimise eight parameters in the model such as the size of the neighbourhood and the production costs. By running the GA several times and examining the variation in the parameters, it was possible to compare which parameters were close in value to those originally determined by the modeller using knowledge of petrol markets and which parameters varied considerably between runs and therefore had little effect on the overall results of the simulations.

More recently, Stonedahl (2011) undertook a comprehensive evaluation of GAs for parameter estimation of ABMs in a number of applied areas including archaeology and viral marketing, which showed that GAs can be effective tools for uncovering and further investigating interesting behaviours in these applied areas. However, the research also recommended experimentation with further applications as well as a consideration of multi-objective optimisation problems. The aim of this chapter is to provide an example of single-objective GA parameter estimation in another application area, i.e. crime. An ABM of burglary, which has been previously developed and applied to the city of Leeds in the United Kingdom (Malleson et al., 2010a, b, c), is used to examine the use of a GA for parameter estimation. The chapter begins with on overview of basic crime theory and previous modelling research, including why ABMs are well suited to modelling criminal behaviour. This is followed by a brief overview of optimisation methods and the basic mechanism of a GA. The ABM of burglary is then described including the parameters to be optimised and the GA experimental settings. This is followed by the results of some preliminary experiments and initial reflections upon this method for parameter estimation. The chapter concludes with plans for further research in this area.

2 Theoretical Background

Individual acquisitive crimes are the result of the convergence of a huge number of factors. These include, but are not limited to:

- The motivation of the offender;
- The behaviour of other people including the victim(s);
- The influence of the surrounding physical environment;
- Wider social factors such as levels of community cohesion.

Each of these elements are also extremely complex in their own right. The motivation/behaviour of the offender and other people depends on a wealth of complex psychological characteristics and life experiences as well as factors such as daily routines and transport networks that put people in a particular place at a particular time. The physical environment contains a broad range of ‘cues’ that might encourage or deter crime (such as high hedges that block visibility, burglar alarms, building security, etc.) -- identifying these cues and their impact on offenders is non-trivial. Wider social factors also have a direct influence such as determining how comfortable an offender feels in a particular area (i.e. whether or not they stand out) as well as broader effects that influence where people travel to within a city.

Although the system is clearly complex in the scientific sense of the word, occurrences of crime are not random. Crime patterns can remain stable over long periods of time and a large body of literature has evolved to explain them. This section will outline some of the most relevant criminological findings which form the basis of ABMs of crime as introduced in section 3. As well as demonstrating that the model closely reflects the reality of the real-world crime system, it will make clear why the ability of agent-based modelling to account for the behaviour and interactions of numerous individuals makes it the most suitable methodology for modelling acquisitive crime and burglary in particular.

2.1 The Spatial Scale of Crime Analysis

Over time, research that seeks to understand the spatial patterns of crime has been moving progressively towards the use of smaller and smaller geographies. In their seminal work on juvenile delinquency, Shaw and McKay (1942) used the census tract (an American administrative zone of approximately one square mile). This is roughly the unit of analysis that most modern crime research has continued to use (Weisburd et al., 2009), with the exception of some more recent studies that work at smaller census area boundaries of approximately 100–200 households. However, modern environmental criminology theories and recent empirical research (Weisburd et al., 2004; Andresen and Malleson, 2011) suggest that even the smallest areal units of analysis (such as census output areas of less than 1000 people) hide important intra-area crime patterns. As a result of these discoveries, a movement in Environmental Criminology began which focused on the 'micro-places' in which crime occurs (Eck and Weisburd, 1995). For example, burglars choose individual homes based on their individual characteristics (Rengert and Wasilchick, 1985) so it cannot normally be assumed that a community or neighbourhood is homogeneous with respect to burglary risk. Similarly, recent work on repeat-victimisation (e.g. Johnson *et al.* 2007) has identified extremely tight spatio-temporal clustering around individual burglary victims. These findings are particularly relevant as most crime modelling research uses aggregate data that hide these important micro-level patterns (see section 2.3 for more details).

2.2 Environmental Criminology Theories

The movement in crime research towards using individual-level geographies also resonates with the major theories in Environmental Criminology. As this section will illustrate, these theories focus specifically on the spatio-temporal behaviour of the individual(s) involved in crime events and the intricacies of the immediate surrounding physical environment.

Routine activity theory (Cohen and Felson, 1979) explores the interactions between victims, offenders and other people who might influence an individual crime event (e.g. passers-by, police, etc.). For the crime to occur, the theory stipulates that an offender must meet a victim at a time and place with an absence of others who might prevent the crime. This convergence depends on the routine activities of the people involved. For example, a burglar might come into contact with a vulnerable house (the potential victim), but might not be able to commission a crime if the routine activities of the residents or neighbours mean that they are in the area at the same time and will notice a crime taking place.

The geometric theory of crime (Brantingham and Brantingham, 1981) shares many similarities with routine activities theory, but focuses more explicitly on the interdependencies between a person's knowledge of the environment, i.e. their *awareness space*, and criminal opportunities. The theory considers how the routes used to travel around a city influence a person's awareness space and hence the spatio-temporal locations in which offenders are likely to commit a crime. Burglars do not search for targets at random; instead they are likely to search near important 'nodes' such as friends' houses, schools, work places, or places of leisure (Brantingham and Brantingham, 1993). Thus house vulnerability to burglary is less relevant if the house itself is not within the awareness space of a person who might attempt to burgle it.

The final theory that the model logic attempts to replicate is the rational choice perspective (Clarke and Cornish, 1985). This suggests that the offender's decision to offend is a cost-benefit analysis weighing up potential rewards of a successful crime with the risks of being apprehended. Thus a crime will only be committed if it is perceived as profitable. It is important to view the concept of rationality as 'bounded', such that a decision that might appear to be optimal to one person (in a specific situation with their own thoughts and motivations) might be blindingly irrational to another.

Although they describe different elements of the crime system, the theories largely agree on the mechanisms that lead to the spatio-temporal patterns of crime. A factor that is particularly relevant to crime modelling is that in each theory the emphasis is on the *individual-level nature* of crime occurrences. The crime system is driven by the behaviour and interactions of individual people situated in a highly detailed local environment. Aggregating such a system (either spatially or temporally) will hide important lower-level dynamics that ultimately explain why crime takes place in the places that it does.

2.3 Traditional Crime Models

Traditionally, quantitative crime models have used area-based crime data in regression style modelling (see e.g Brantingham and Brantingham, 1998). Kongmuang (2006) provides a comprehensive review of the methods employed, where a number of common characteristics can be identified. For example, model accuracy is usually estimated through the Akaike Information Criterion (AIC) or a goodness-of-fit statistic such as R^2. Other drawbacks are outlined below although we do recognise that there are also many advantages of statistical methods which are not discussed further in this chapter.

Firstly, statistical models generally utilise simple functional relationships, e.g. they cannot adequately capture the evolution of individuals through time and the effect this has on their behaviour. In contrast, ABMs can represent these complex real world interactions including the intricate personal trajectories and histories of individuals. Statistical techniques generally aim to reduce variables to enhance explanation at a cost to predictive power, so cannot account for the complexity of the environmental backcloth and the non-linear human-human or human-environment interactions that drive the system.

Secondly, the use of spatially aggregated data -- to represent crimes, demographics, the environment, etc. -- hides important lower-level relationships between crime, individuals and the environment. Similarly, it is not possible to capture important features of the physical environment such as accurate travel times, impassable barriers or road-network layout unless individual environment objects (roads, buildings, parks, etc.) are accounted for explicitly.

Finally, linear models may be “computationally convenient” (Eck and Liu, 2008), but they cannot represent the dynamics of complex systems. Complex systems are driven by the behaviour of and interactions between the individual components of the system. These fundamental drivers of the system are lost when the underlying data are aggregated.

In general, the dynamics that drive the *crime system* (as with other social systems) are not captured directly in aggregate models. This makes it difficult both to explore criminology theory -- which inherently focuses on the spatio-temporal behaviour of individual people -- and to make crime forecasts at the same time. ABMs, however, provide an alternative approach by allowing these individual entities to be modelled *directly*. In this manner, it is possible to capture the true richness of the system and much more closely reflect an individual's unique circumstances and behavioural characteristics.

3 Agent-Based Models (ABMs) of Crime

ABMs have a number of clear advantages over other modelling and analysis techniques when it comes to understanding crime. Crime tends to be the result of individuals acting on the basis of their history and current environment, either alone or in collaboration. ABMs, unlike other techniques, take as their starting point unique individuals ('agents') with their own history and decision making capacities, and these individuals are placed in a complicated environment to discover the resultant behaviour. As in a real crime system, agents will both respond to and adjust the current environment (e.g. agents may cause an area's attractiveness to housebuyers to fall). The agents in ABMs can interact and collaborate in group behaviour and decision making. However, there is also no reason why larger, aggregate groupings and decision making (e.g. government policy groups) cannot also be represented and respond to the system. In short, ABMs represent social systems in the way we intuitively understand social systems ourselves.

This does not, of course, *necessarily* make such models better ways of understanding such systems. However, in practice there are considerable advantages to matching our understanding of reality as closely as possible. Firstly, ABMs allow for the direct representation of decision making using rulesets that act at the individual level. This means that the errors associated with representing behaviour are less likely than, for example, if such behaviours were represented as aggregate mathematics. The concentration on rulesets and behaviour also means that ABMs can act as a framework for the representation and testing of qualitative social theory described at the individual level, something much harder to achieve with mathematical or statistical representations. ABMs act as a framework for understanding emergence, that is, how behaviour at the individual level can generate complex patterns at some larger scale (like crime hotspots). Secondly, agents can have an individual history. Statistical techniques are limited in their ability to track how life-events and the environment interact. While it is possible to run statistical microsimulations that look at the results of such interactions, after multiple events and with low populations, these techniques become problematic. With ABM, individuals carry their history with them, either implicitly or explicitly, and this history can be analysed to see how it affects their decision making. Finally, ABMs can represent a wide range of environments, from the very abstract, to the extremely realistic. This allows us to explore and understand the effects of the environment on behaviour at a very detailed level. For example, it is possible to look at the effect that a specific change in a public transport route might have on criminal opportunity. Moreover, once an ABM is set up, a wide variety of different analyses and scenarios can be run without adjusting the underlying model, unlike many other techniques, where the model must be specifically designed from the ground up to answer a single research question.

Given these advantages, it is somewhat surprising how slowly the development of ABM of crime has progressed. Nevertheless, the last ten years or so has seen an increased interest in the technique, and a number of groups are building ABMs of crime of various levels of detail. In general, most current ABMs of crime attempt to replicate the major components of the criminal system to some degree: offender motivation and decision-making, offender behaviour and movements, victimhood and guardianship. However, given the complexity of the system, it should come as no surprise that most concentrate on building realism in one of these components rather than all of them. In addition, the realism of the environment within the model varies a great deal, not least because of the broad division in agent-based modellers between those who believe ABM should be utilised as abstract 'thought experiments' to explore key theoretical behaviours and ideas, and those who believe that it is possible to build a more detailed model of the real world for exploration and prediction (see, for example, Di Paolo *et al.*, 2000, for arguments for the former).

Malleson *et al*. (in press) give a full review of ABMs used to model crimes that have a predictable geographical component (that is, crimes like burglary and street theft, as opposed to crimes like domestic violence and fraud, on which geography have less obvious effects). However, notable models at the more abstract end of the scale include Winoto (2003); van Baal (2004); Brantingham and Brantingham (2004; Brantingham *et al*. 2005a;b; 2008); Dray *et al*. (2008a), and Wang *et al*. (2008), while more realistic models have been attempted by Liu *et al*. (2005), Melo *et al*. (2005), Birks (2005; 2007, Birks *et al*. 2008, 2012) Malleson *et al.*, (2009; Malleson, 2006); Groff (2006; 2007a;b; Groff and Mazerolle, 2008), and Malleson (2010; Malleson *et al.* 2010a; b; c), and on social, but not geographical realism, Hayslett-McCall *et al.* (2008). In addition, the technique is seeing a growing use in modelling crimes where geography is secondary to social organisation, e.g. gang crime and civil violence (Lustick 2006; Huddleston *et al.* 2008; Bhavnani *et al.* 2008; Cherif *et al.* 2009; Egesdal *et al.* 2010). Some of the ethical issues facing agent-based modellers of crime are explored by Evans (2012).

4 Optimisation of Complex Models

We now consider the specific issue of parameter estimation in ABMs using methods of optimisation, which are techniques that search a problem space for the best solution possible given the complexity of the problem, the computational resources available and the objectives or constraints of the problem (Goldberg, 1989). Mathematically this involves finding what are referred to as a set of decision variables that minimise or maximise one or more objective functions (i.e. a function specific to the problem which determines how good the solution is) subject to satisfying a set of constraints. For example, the decision variables may be the quantity of material that flows between a set of different distribution points where the objectives are to minimise the distance travelled while maximising the profit subject to certain routes not being allowed due to direction of flow or excessive gradients. Some problems may have a single optimal solution where the main challenge is finding the global optimum in a solution space characterised by multiple local minima (or maxima depending upon the way the problem is formulated) without having to fully search the whole parameter space. In contrast, in more complex problems or in those with multiple objectives, there is no single solution that simultaneously optimises all conflicting objectives. The result is a set of alternative optimal or feasible solutions of similar fitness that represent trade-offs between the different objectives. Optimisation provides the mechanism to find this set of solutions, which are called Pareto optimal solutions. Other methods, such as multi-criteria decision making, are then needed to further evaluate the solutions that are identified during the optimisation process. Many real world problems are characterised by the need to take conflicting multiple objectives into account. In hydrology, for example, multi-objective optimisation methods are used extensively for calibrating physical and conceptual hydrological models (Yapo et al., 1998; Vrugt et al., 2003; Efstratiadis and Koutsoyiannis, 2009).

Optimisation can be divided into the following seven steps: identify the parameters in the problem or model; choose the design variables (or those which require optimisation) from this set of parameters; outline any constraints which must be taken into account during the optimisation; choose appropriate objective functions, i.e. methods of evaluating the solution or model performance; set the allowable range for the decision variables; choose an appropriate optimisation algorithm; and run the algorithm to obtain the results (Deb, 2001).The next section deals specifically with step 6, i.e. different methods of optimisation.

4.1 Methods of Optimisation

A number of different optimisation methods have been developed in the past to handle problems involving single and multiple objectives. Classical (or conventional) optimisation methods were developed using differential calculus. They involve finding an analytical solution on functions that are continuous and differentiable, e.g. the simplex method (Maros and Mitra, 1996). These are referred to as strong methods and are deterministic. On the other end of the spectrum are weak classical methods, which involve a random or stratified random sampling of the search space in order to find the solution, which are inefficient methods. These classical methods have a number of disadvantages (Goldberg, 1989; Deb, 2001). For example, the convergence of an optimal solution depends upon the initial solution and they are not efficient for problems with discrete rather than continuous search spaces. Moreover, they are not efficient in solving non-linear, complex problems with large search spaces and many conflicting objectives and they cannot be parallelized efficiently since they use a single search path to obtain the optimal solution. For this reason, a set of intermediate methods have been developed that contain a stochastic element and which use more effective search strategies to avoid being trapped in local minima, e.g. simulated annealing, tabu search and evolutionary methods such as genetic algorithms (GAs). The focus of this research is on GAs, which are described in more detail in the sections that follow.

4.2 Genetic Algorithms (GAs)

GAs are intrinsically suited to optimisation when the fitness landscape is complex, changes over time or has many local optima. Through inherent parallelism, they are able to simultaneously explore numerous potential solutions (Holland, 1992; Mitchell, 1998; Goldberg, 1989). Within a GA, data are represented as binary strings or the individuals that make up the population. These individuals are also referred to as chromosomes where each chromosome is comprised of genes (or the variables in the string). Through the evaluation of the fitness of one string, a GA is also simultaneously sampling each of the many other spaces to which it belongs. Over several fitness evaluations, the GA builds up an increasingly accurate value of the average fitness of each of these spaces. Through the evaluation of a small number of individuals, a much larger group is being evaluated implicitly. By this mechanism, a GA can 'home in' on the space with the highest-fitness individuals. This combination of parallelism, along with the other major components of a GA which produce the evolution of fitter solutions, i.e. selection, mutation and crossover, make this approach a very powerful and efficient tool.

GAs follow the same basic set of steps as outlined in Figure 1. A population is first initialised and the objective functions are then set. The fitness of each individual is assessed and on the basis of this, the fittest in the population are selected for reproduction via crossover. This continues over many generations or iterations until predefined criteria are satisfied, e.g. a certain threshold value for the objective function has been reached. For a more detailed overviews of GAs, the reader is referred to Goldberg (1989), Davis (1991), Michalewicz (1992), Bäck and Schwefel (1993) and Eiben and Smith (2003). For an overview of GAs in the context of geographical optimisation, Xiao (2008) provides an excellent introduction.

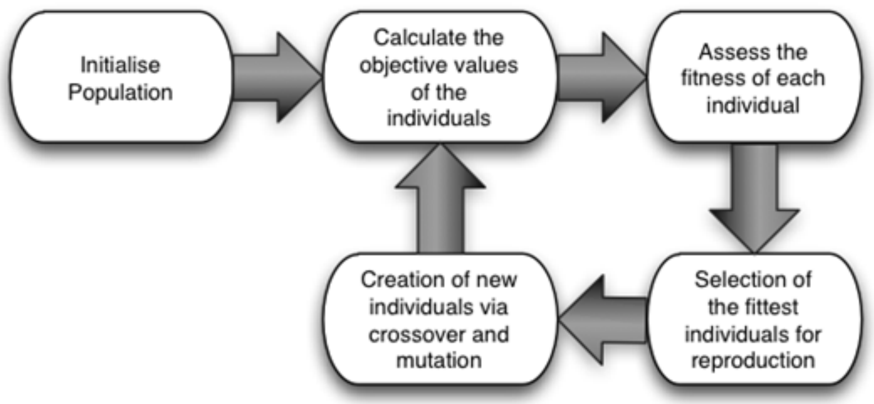

Fig. 1. The basic operation of a GA (adapted from Weise, 2009).

The following sections will briefly outline the main generic parameters and processes that all GAs share.

4.2.1 Initial population

At the start of an optimization, the GA requires a set of initial solutions. There are two ways of forming this initial population. The first involves randomly generating solutions while the second uses some expert knowledge about the problem or another method to generate this initial population. The advantage of the second method is that the GA starts with a set of approximately known solutions and therefore may converge to an optimal solution faster than the first method. The disadvantage is that genetic diversity may be restricted and limit the ability of the GA to generate optimal solutions that might only be arrived at through a random starting position.

4.2.2 Representation

Most of the problems suitable for GAs involve identification of a set of parameters that need to be represented in such a way as to allow evolutionary operators to be effectively applied. As GAs are robust, there is little need to rigorously identify the 'best' representation for a particular problem (Goldberg, 1989). There are two broad methods that can be used for representation: binary alphabets (Holland, 1975) and real numbers (Davis, 1991; Beasley et al., 1993; Michalewicz and Janikow, 1991; Michaelewicz, 1992). There is no single 'correct' coding method for encoding a problem; the mode of representation is dependent on the problem. However, the coding sequence must adequately represent the problem to ensure that the optimal solution is available to the algorithm and be bounded by an allowable range for the parameters.

4.2.3 Fitness and Selection

In order to evolve better performing solutions, the fittest members of the population are selected and randomly exposed to mutation and recombination (as described below). This produces offspring for the next generation. The least fit solutions die out through natural selection as they are replaced by new recombined, fitter, individuals. Evaluation of the fitness of the individuals involves some form of comparison between observed and model data, or a test to see if a particular solution meets pre-defined criteria or constraints. In this work, the Standardised Root Mean Square Error (SRMSE - Knudsen and Fotheringham, 1986) is used to estimate the difference between real crime data and the model results.

There are number of possible ways for selection to take place and Table 1 describes the main parental selection schemes that recur within the literature.

Table 1. Description of several of the most common forms of parental selection

Selection Type	Description
Ranking	The population is sorted from best to worst. The number of copies that an individual receives is given by an assignment function and is proportional to the rank assignment of an individual.
Tournament	A random number of individuals are selected from the population. The best individual from this group is chosen as a parent for the next generation. This process is repeated until the mating pool is filled.
Roulette Wheel	Individuals are mapped to contiguous segments of a line, such that each individual's segment is equal in size to its fitness. A random number is generated and the individual whose segment spans the random number is selected. This process is repeated until the desired number of individuals is obtained.
Truncation	Truncation sorts individuals according to their fitness (from best to worst). Only the best individuals are selected to be parents.

4.2.4 Selection pressure

Along with the selection method, the selective pressure parameter is critical. This parameter measures the probability of the best individual being selected compared to the average probability of selection and drives the algorithm towards a solution. The value for this parameter should be carefully selected as too much selective pressure can lower the diversity within the population resulting in sub-optimal solutions. Conversely if the selection pressure is too low, the population remains too diverse and the optimal solution is not found.

4.2.5 Recombination/Crossover

The main reproductive genetic operator is recombination (also known as crossover). This is the process by which new individuals are produced by combining the information from two parent chromosomes. The resulting offspring inherits components from both parents (Figure 2). This allows the EA to explore new areas in the search space. Without recombination, the offspring are simply duplicates of the parents, which does not provide the opportunity to improve the fitness of the population.

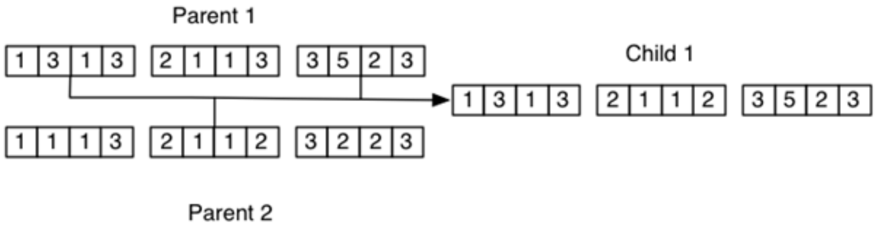

Fig. 2. Representation of recombination between two parents to produce an offspring

There are several methods of recombination available; the suitability of the method is dependent on the types of genes or variables stored in the chromosome. The three most common approaches are intermediate, line and extended line recombination methods. In intermediate recombination, the variable values of the offspring are randomly chosen from between the values of the parents. Normally values of up to 25% outside this range can be used, which has been chosen to ensure that statistically a space covered by the recombination does not decrease in size with time leading to a loss in diversity. The position of the variable chosen on the line determines how much each parent contributes to the offspring and is chosen uniformly at random for each gene. Line recombination is similar to intermediate recombination except that the same random number is used for selecting the value of every gene in a chromosome. Extended line recombination is different from the above techniques in that the variable range is not limited to a range around the parents. The probability of any particular value being taken is not uniform but varies with a high probability near the parents and a low probability far away from the parents. The probability distribution can also be chosen to favour the fitter parent. The value controlling the amount of the parent that is used is generated randomly and then used for selecting the value of subsequent genes.

4.2.6 Mutation

The process of recombination can produce a very large number of new individuals. However, if the GA is moving towards an optimal solution (and hence a smaller population pool), it is possible that the available solutions are suboptimal. Through the alteration of one or more parts of the chromosome, mutation introduces diversity into the selected population which can potentially breed fitter solutions (Figure 3). The mutation rate is generally a random probability determined by initial experimentation.

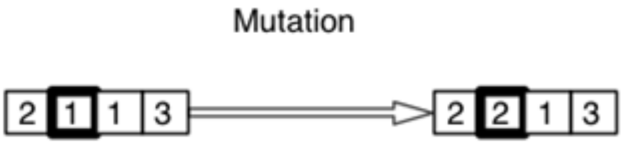

Fig. 3. Illustrating the mechanism of mutation

The literature offers no strict guidelines for the selection of the size of the mutation step. The optimal step-size depends on the research problem and may even vary during the optimisation process. Small mutation steps are acknowledged in the literature as being successful, especially when the individual is already well adapted. However, large mutation steps can, when successful, produce good results very quickly. A good mutation operator should therefore produce small step-sizes with a high probability and large step-sizes with a low probability.

In the next section, the ABM burglary model is introduced along with the settings of the GA for parameter estimation.

5 The Agent-Based Burglary Model

The model utilised here attempts to provide a detailed burglary model at the city scale that includes a) detailed offender drivers, decision making, and behaviour; b) realistic victim distributions and attributes, including daily variations in household occupancy; and c) a realistic environment including a full transport network and reasonable levels of guardianship, including community guardianship. A full ODD protocol (Grimm and Railsback, 2005; Grimm et al., 2006) description of the model can be found in Malleson et al. (in prep), while a detailed description of the model design and data preparation is given in Malleson (2010). The full model uses the PECS framework (Schmidt, 2000; 2002; Urban 2000) for internal offender decision making can be found in Malleson et al. (2010b). However, to simplify the model for the application of the GA, a simpler behavioural framework was implemented, the details of which are outlined below. Calibration and validation of the model were carried out manually in the past; details can be found in Malleson et al. (2010c). Here, we briefly describe the model in sufficient detail (using a simplification of the ODD protocol) to understand the broad model workings and how the parameters that will be calibrated fit into the model.

5.1 Purpose of the Model

The motivation behind the model is to simulate the spatio-temporal locations of burglaries at the city scale and, ultimately, to provide a framework for modelling and testing our understanding of the criminal system. The model runs for a fixed length of simulated time -- sufficient to reach dynamic equilibrium -- so does not predict the actual number of crimes. Instead, we focus here on the values of the behavioural parameters that drive the behaviour of the agents to determine what these tell us about the behaviour of burglars in the real world.

There is no notion of the processes that lead to someone 'becoming' a burglar; each agent has only one purpose which is to commit burglary. In addition, there is no notion of punishment or capture – offenders are not removed from the system, nor are their drivers adjusted by any kind of punishment. Although variables such as community guardianship help to determine whether a property is chosen for a crime, a chosen target is always successfully victimised. There is also no communication between agents; all offenders are currently lone individuals without a shared understanding.

The model generates a spatial distribution of crimes, taking into account a variety of offender behaviours, environmental factors, and victim and guardian attributes.

5.2 Data and the Study Area

The study area for this research covers 1,700 hectares in the city of Leeds, UK. The area contains some of the most deprived neighbourhoods in the country and was earmarked for an ambitious urban renewal scheme which makes it an ideal candidate for predictive crime modelling. Figure 4 illustrates the data used to represent the study area in the simulation.

- *Communities* are generated using the Output Area geography (a census area boundary containing approximately 100 houses) and classified using the Output Area Classification (OAC: Vickers and Rees, 2007). This allows community types to be compared quantitatively.
- The home locations of *offenders* were estimated from police recorded crime data on convicted burglars.

- *Roads* and *buildings* are established from Ordnance Survey MasterMap data (the Integrated Transport Network and Topographic Area data sets respectively)
- *Expected data* are required to validate the model. The data used here are the number of burglaries per Output Area that occurred in 2001. This year was chosen because it corresponds closely with the timing of the UK census from which community demographics are estimated. As mentioned in section 4.2.3, the SRMSE is used to compare model results to expected data at the output area level.

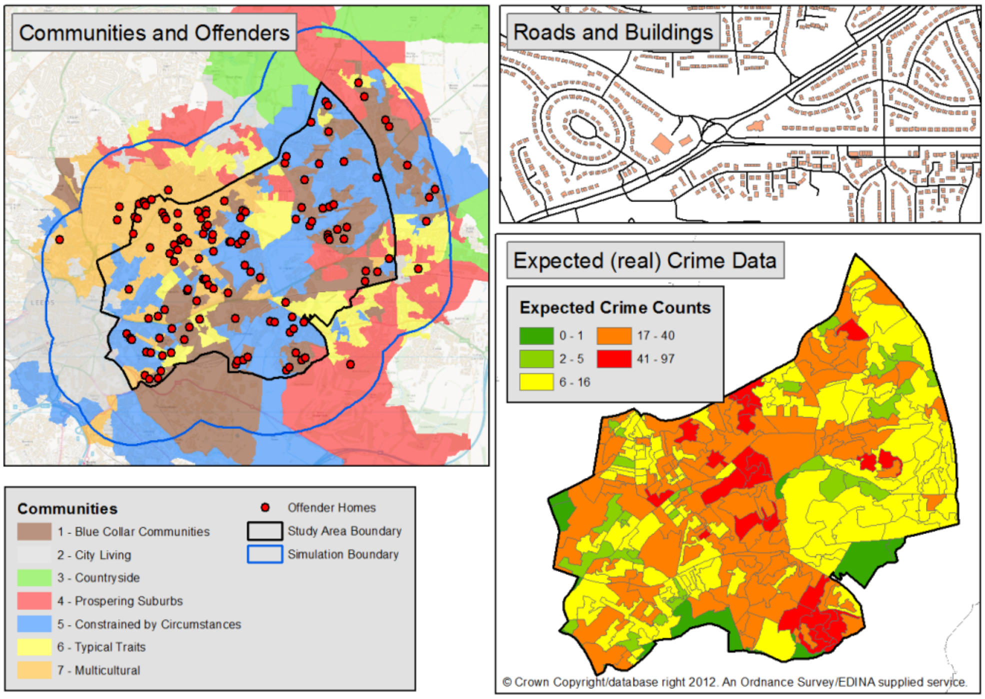

Fig. 4. Datasets used in the ABM of burglary

Finally, a buffer zone is used to reduce the boundary effects in the results. Thus all burglaries that occur outside the study area boundary are discounted when simulated burglaries are compared to real data.

5.3 State Variables and Scales

5.3.1 The Agents

The model is comprised of agents representing offenders. Victims and guardians are represented through environmental variables, e.g. the estimated level of community cohesion. Each offender agent is assigned a home building (and associated community) at model initialisation. This location, derived from real offender data, is where the agent lives. The main agent variable, which changes during the model run, is the *burglary motive*. This variable increases over time and determines whether or not a burglar will choose to target an individual house; more details follow in section 5.3.3. Once a burglary has been committed, this level falls to zero.

5.3.2 The Environment

Objects within the environment build up a substrate in which the agents act. There are three types of objects:

- **Roads** are used to restrict the possible spatial locations of the agents. In the full version of the model, roads can be used to simulate different transport speeds and routes (e.g. a car driver moving faster along a major road) but in this simplified version all agents move at a constant speed of 4 miles per hour (a fast walking pace).
- **Buildings** represent the houses in which agents live and also represent the potential victims of burglary. Their spatial locations and attributes have been established from the Ordnance Survey MasterMap geographic data product.
- **Communities** represent the neighbourhoods in which the houses are located. They will influence whether or not a burglar chooses a particular house and also determine where an offender starts their search.

Section 5.2 outlined the data sources that have been used to generate these layers while Table 2 summarises the fixed attributes for the buildings and communities. As section 5.3.3 will illustrate, these will influence an agent's burglary decision about where to look for victims and which individual building to target.

Table 2. Parameters associated with communities and buildings

Parameter (abbreviation)	Description
COMMUNITIES	
Attractiveness (ATT)	A measure of the abundance of valuable goods that is likely to be found in houses within the community. This measure was calculated from factors such as the percentage of full time students and the percentage of houses with more than two cars.
Social Type (SocT)	A vector containing the 41 different OAC variables. This can be used to compare the Euclidean difference between two communities.
Collective Efficacy (CE)	A measure of the cohesion of the community, calculated from a combination of deprivation, residential stability and ethnic heterogeneity.
BUILDINGS	
Accessibility (ACC)	An estimate of how easy a building is to enter based on its spatial properties. Houses with many exposed walls are assumed to contain a larger number of doors or windows to provide access to a burglar.
Visibility (VIS)	A measure of how visible a building is to its neighbours and to passers-by. This is calculated from the size of any attached garden and the number of additional buildings within a buffer zone.

5.3.3 Process Overview and Scheduling

The model is initialised with data that allocates offenders to households, attributes to buildings and transport components, and initialises the state variables of the offenders. Offenders start with nothing in their awareness space. At each time step, all offenders decide on actions determined by their internal states. The sequence of offenders is random.

In its full implementation, the model uses an advanced behavioural framework to equip the agents with realistic daily behaviour including sleeping, using substances and socialising. However, the focus of this work is to better understand the relationship between the behavioural parameters and simulated burglary locations so the behaviour of the agents has been simplified by removing non-burglary activities. Therefore, activities such as socialising or using substances will not have an influence on the final locations of the burglaries. As mentioned previously, each agent is driven by a single ‘burglary motive’, which increases over time until a burglary is committed. A burglar’s behaviour schedule is as follows:

1. At 09:00 simulated time, the agent chooses a community to travel to in search of a burglary target. The following formula is used to assign the likelihood, L, of choosing each community, a, relative to their current location, c, and their home community, h:

$$L = w1 * (1/\mathrm{dist}(c,a)) + w2 * \mathrm{Attract}(h,a) + w3 * \mathrm{SocialDiff}(h,a) + w4 * \mathrm{PrevSucc}(a) \quad (1)$$

where dist(c,a) represents the distance (travel time) from their current location to the target community, Attract(h,a) represents the relative attractiveness of the community compared to the agent's home area, SocialDiff(h,a) returns the similarity of the target community and the agent's home (where similar communities are favourable) and PrevSucc(a) returns the number of times an agent has had a successful burglary in the past. Importantly, the weights $w1$ to $w4$ can be used to assign an importance to each factor -- if a weight is large then the parameter will have a greater influence on the agent's behaviour. Roulette-wheel-selection is used to randomly choose a community from all of those available such that those with a greater likelihood value have a greater chance of being chosen.

2. On the way to their destination, the agent observes each house that they pass and a 'risk' for burglary is calculated as follows:

$$R = (w5 * CE + w6 * ACC + w7 * VIS) / (w5 + w6 + w7) \quad (2)$$

where CE is the perceived collective efficacy of the surrounding area, ACC represents the accessibility of the target building (how easy it is to enter) and VIS represents the visibility of the building to neighbours and passers-by (where high visibility increases the risks). If this risk value is lower than the agent's current burglary motive, then the agent *might* commit a burglary - the probability of actually committing a burglary increases exponentially as the difference between risk and motive increases. Again, weights applied to each parameter determine how much of an influence each factor will have over the agent's decision.

If the burglary is successful, then the agent travels home. In this manner the model has been configured to allow for a variation in offending behaviour (the same agent will not always choose the same house) but agents will, on average, always commit one burglary per day.

3. If the agent reaches their chosen destination without committing a burglary, then they repeat the process. This continues until a victim has been found.

6 Exploring the Dynamics of Criminal Behaviour

The model outlined in section 5 is an advanced ABM that attempts to closely represent criminological theory and the experiences of crime-reduction experts in the field. There are 7 different variables that determine where burglar agents will start searching for targets and which houses, in particular, they will actually victimise. Although some experimentation with changing the parameters can be undertaken manually through trial-and-error, the number of combinations that can be tested, even with 7 parameters, is limited. Thus, the GA provides a more comprehensive approach to more intelligently explore the entire 7-dimension parameter space. This can help to determine which parameters have the most substantial influence on the model, and the values may eventually inform our understanding of the behaviour of burglars in the real world.

A set of preliminary experiments have been undertaken here using a GA to find the values of the 7 parameters. A population of 20 individuals was used and the GA was tracked over three iterations. The fitness of each model configuration (or 'chromosome') is provided in Figure 5, which is plotted against the iteration number. As would be expected, the GA is able to identify which model configurations result in the lowest error and, hence, which should be used to generate the configurations in the next iteration. Accordingly the model error decreases with each subsequent iteration. Figure 5 also highlights some clustering of fitness values after the initial (random) population undergoes an evolution. This illustrates that the algorithm is fine-tuning the ABM in different parts of the parameter space that have the lowest error.

Table 3 provides the values for each of the parameters for the models with the lowest error after each iteration. The GA appears to converge very quickly to an optimal configuration, which is found after the first iteration and does not change substantially over the next few iterations, although a marginally different configuration is found in iteration 3. This implies that the algorithm has found a global maximum. Alternatively, the model may need to run many more generations, but as discussed in the next section, the computational load in using a GA for parameter estimation of this ABM was too high to allow for further experimentation at this stage.

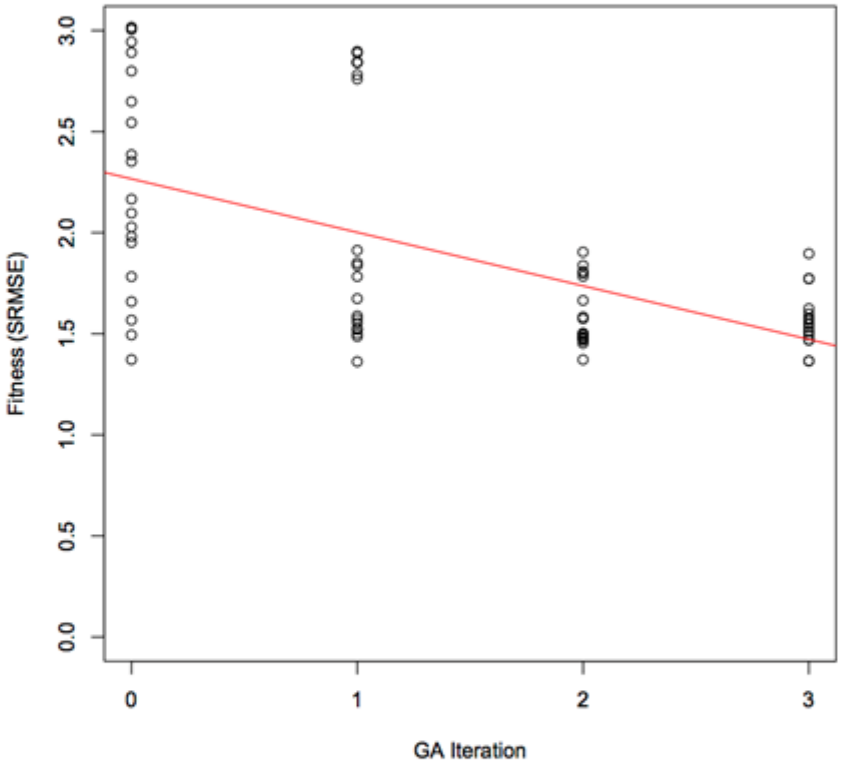

Fig. 5. Fitness of all the chromosomes by GA iteration

Table 3. Values of the parameters after each iteration

Iteration	Fitness	w1	w2	w3	w4	w5	w6	w7
0	1.372	0.719	0.668	0.736	0.683	0.541	0.291	0.984
1	1.362	0.719	0.668	0.736	0.683	0.541	0.291	0.984
2	1.372	0.719	0.668	0.736	0.683	0.541	0.291	0.984
3	1.365	0.689	0.717	0.781	0.727	0.510	0.241	1.000

The weights: w1 = distance; w2 = attractiveness; w3 = SocialDifference; w4 = PreviousSuccess; w5 = CollectiveEfficacy; w6 = Accessibility; w7 = Visibility

Figure 6 illustrates the change in parameter value for these best models. Interestingly, the Visibility parameter is consistently assigned a high value, which implies it is an important factor in the agent's decision compared to other variables. Conversely, Accessibility appears to have only a marginal importance. Explanations for these findings will be discussed in the following section.

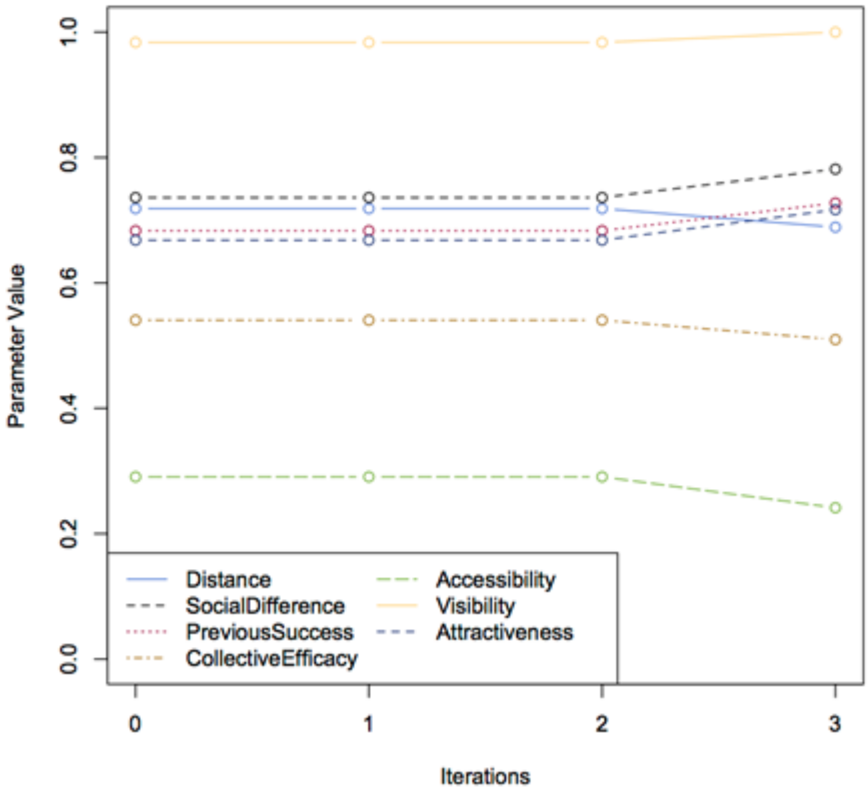

Fig. 6. Parameter values for the best model after each GA iteration

To review the results spatially, Figure 7 presents maps of the burglary counts generated by the three best model configurations after GA iterations one ('Model 1), two ('Model 2') and three ('Model 3'). The results are presented in two forms. The maps on the left hand side of Figure 7 present results that have been spatially aggregated to the geography of the communities (i.e. each community has a count of burglaries that were committed during the model run) and the maps on the right hand side of Figure 7 present point density estimates produced using the Kernel Density Algorithm (KDE). The use of KDE arguably presents a more accurate picture of the underlying point patterns and is commonly used by police analysts (Chainey and Ratcliffe, 2005).

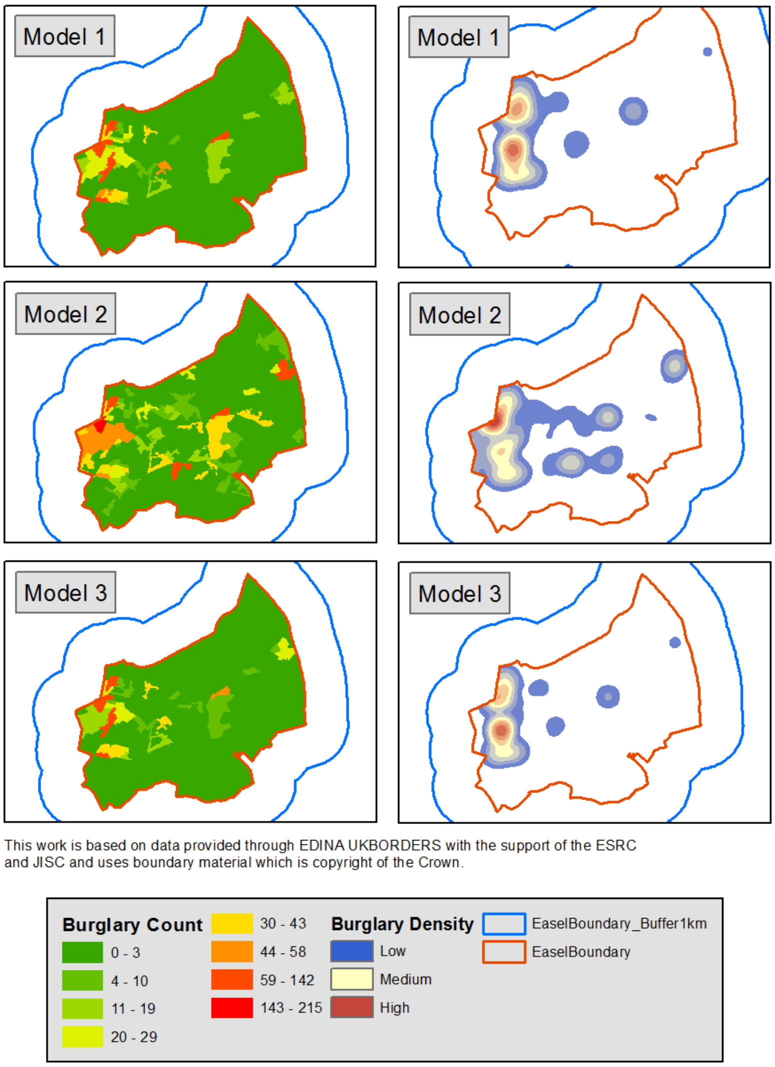

Fig. 7. Results from the three best model iterations after 1, 2 and 3 iterations

The model results show consistently high numbers of burglaries on the western side of the study area, which matches the general pattern exhibited by the observed data. Interestingly, these larger scale patterns are similar regardless of the differences in configuration, which suggests that small changes to any of the agents' behavioural parameters have little effect on the model results. Some small differences can be seen in the centre of the study area where some small hotspots are picked up differentially between the three models. Any discrepancies are likely to be a result of the probabilistic nature of the model (recall that with sufficient computing power each model would have been run a large number of times to calculate an average error value) although there would be scope to investigate what might be generating these differences, e.g. the slight decrease in the distance, collective efficacy and accessibility parameters in 'Model 3' and a slight increase in all the others. However, it is encouraging that small parameter changes have little effect on the model results because, were this not the case, it would be more difficult to be confident in the robustness of the results. This represents another considerable advantage of the application of an optimisation algorithm to this model.

7 Discussion and Conclusions

This paper presented some very preliminary attempts at using a GA to estimate the parameters of an ABM of burglary. These initial findings indicated that the weight associated with the visibility parameter was consistently high. This means that with the models that closely matched the observed crime data, the agents were more likely to burgle houses that were well hidden from their neighbours and passersby. In the ABM used here, this feature was estimated by combining the physical size of a house's garden with a measure of its isolation (i.e. the number of other properties in the immediate surrounding area). Conversely, the accessibility parameter was consistently the least important. This parameter is calculated by estimating the number of exposed walls in each building such that detached houses are at a greater risk of being burgled than semi-detached houses or terraces because there are likely to be a larger number of entry points. Why this building feature appears to be less important, however, is less clear. One of the next steps will be to explore these geographical parameters is more detail.

However, one major issue with the GA approach as utilised in this example was the large amount of computational time required to run the model, even with only three iterations per GA run. The ABM itself is extremely computationally expensive. Even after some simplifications from the original configuration (Malleson *et al.*, 2010b), a single model run still required approximately 10 hours to complete on a normal desktop machine. The results discussed here were generated with the use of a 16-core Intel Xeon E5-2670 ("Sandy Bridge") virtual machine provided through Amazon Web Services (Expósito *et al.* 2013), but even with this hardware, each GA iteration -- with a population of only 20 chromosomes -- required approximately 20 hours to complete. An associated side effect of the computation time is that it is not feasible to run each individual model configuration multiple times, which would be preferable because it would give a more comprehensive assessment of the model error (the simulation is probabilistic so each run will lead to slightly different results). Future work will port the model to more powerful computer systems and run a GA with a larger population for a larger number of iterations and with multiple runs per model configuration. Only then will it be possible to more exhaustively examine the behaviour of the parameters in relation to burglar behaviour in a real-world setting.

The implications of using such an approach when scaling up an ABM to a much larger area, with larger numbers of individuals and with a much larger number of parameters is clearly evident from these preliminary experiments. Ambitious ABM projects such as modelling the entire economy of the United States (Farmer and Foley, 2009) will clearly face major computational challenges in the future. But without methods like GAs, the task of parameter estimation would render such modelling approaches infeasible.

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